Unintegrated parton densities and generelized parton distributions from a light-cone wave function overlap

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- *PDF and GPD definitions and properties*
- *Closer look to the LCWF overlap representation*
- *Exploring a simple model*
- *Conclusions*

in collaboration with D.S. Hwang (Sejong Univ.)

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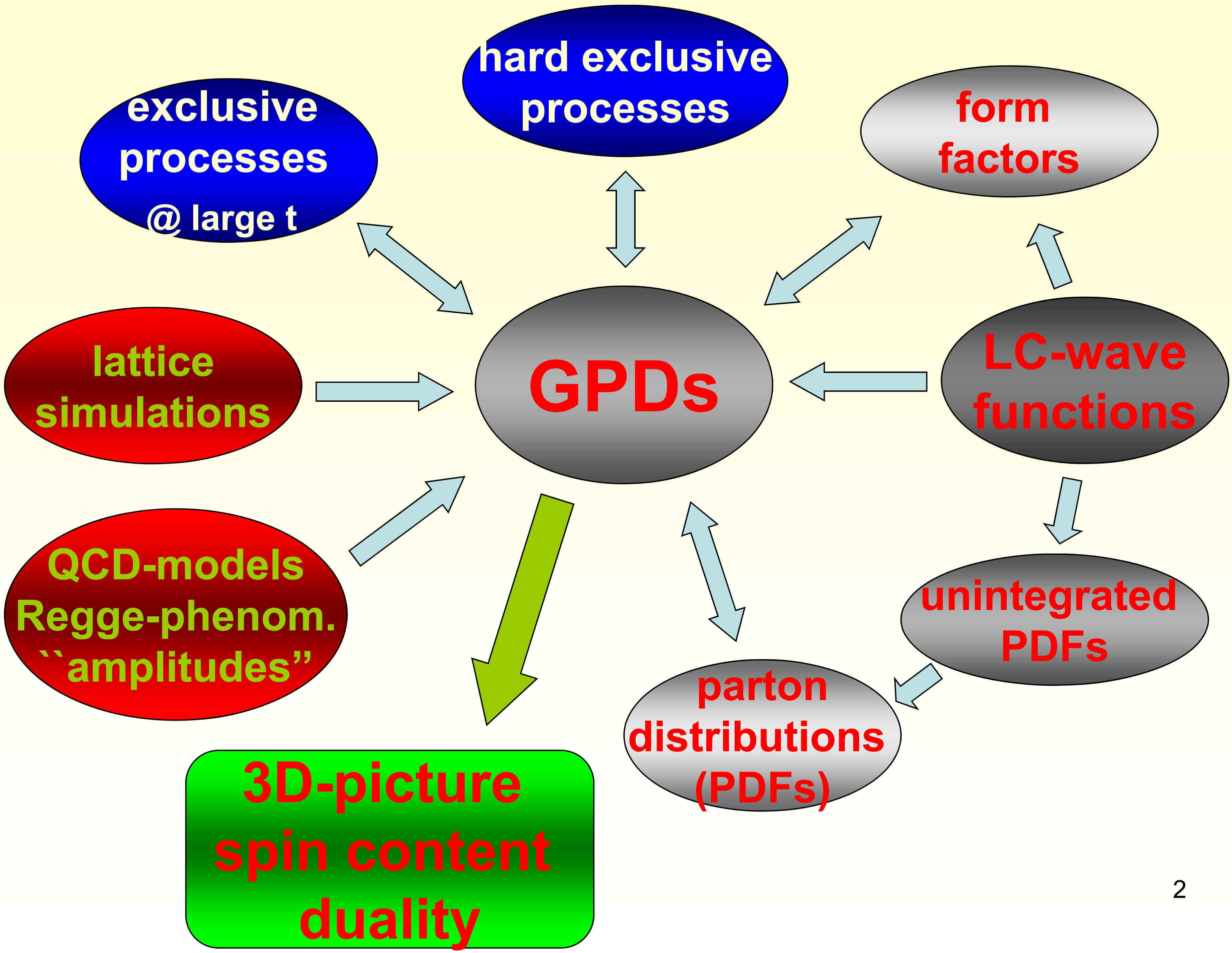
in preparation


exclusive processes @ large t
hard exclusive processes
form factors
lattice simulations
GPDs
LC-wave functions
QCD-models Regge-phenom. ``amplitudes”
unintegrated PDFs
parton distributions (PDFs)
3D-picture spin content duality

Hamiltonian approach to QCD

QCD bound state problem might be formulated in LC quantization:

$$P^-|P,S\rangle = \frac{M^2}{P^+}|P,S\rangle\,, \quad \text{with} \quad P^- = P^0 - P^3\,,\ P^+ = P^0 + P^3\,,\ \mathbf{P}_\perp = 0$$

solution is expanded in *partonic degrees of freedom:*

Drell, Yan (69)
Drell, Brodsky

$$|P, S = \{\uparrow,\downarrow\}\rangle = \sum_{n,\lambda_i} \int [dx\, d^2\mathbf{k}]_n\, \psi^{\uparrow,\downarrow}_{n,\lambda_i}(x_i, \mathbf{k}_\perp, \lambda_i)|n, x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle$$

PDFs, FFs, GPDs **[Diehl et al. (98)]** are defined as LCWF overlap:

$$F(x \geq \eta, \eta, \Delta^2) \propto \sum_{n,\lambda_i} \left(\frac{1-\eta}{1+\eta}\right)^{\frac{2-n}{2}} \int [dx\, d^2\mathbf{k}]_n \delta\left(\frac{x+\eta}{1+\eta} - x_1\right) \psi^{\uparrow *}_{n,\lambda_i}(x'_i, \mathbf{k}'_{\perp i})\psi^{\uparrow(\downarrow)}_{n,\lambda_i}(x_i, \mathbf{k}_{\perp i})$$

struck quark momentum: $x'_1 = \frac{x-\eta}{1-\eta}\,, \quad \mathbf{k}'_{\perp,1} = \mathbf{k}_{\perp,1} - \frac{1-x}{1-\eta}\mathbf{\Delta}_\perp$

NOTE: $x'_1 \to 0$ for $x \to \eta$

The concept of an effective LCWF

- Is the LCWF approach well defined?
- certainly, all LCWFs are coupled by QCD dynamics
- a solution can not be expected in near future
- to stay with the LCWF concept, we introduce an effective one

$$|P,S\rangle = \sum_{h=-1/2}^{1/2} \int d\lambda^2 \int [dX\, d^2\mathbf{k}]_2\, \Psi_h^S(X_i, \mathbf{k}_{\perp i}, \lambda_i|\lambda^2) \prod_{j=1}^{2} \frac{1}{\sqrt{X_j}}\, |\lambda^2, X_i P^+, X_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h\rangle$$

Ψ_h describes the dynamics of the struck quark with spin h

λ^2 denotes some collective degrees of freedom

$\rho(\lambda^2)$ is a probability density:

$$\langle h', \mathbf{P}_\perp + \mathbf{k}'_{\perp i}, X'_i P^+, \lambda'^2 | \lambda^2, X_i P^+, X_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h\rangle$$
$$= \rho(\lambda^2) \prod_{i=1}^{2} 16\pi^3 \delta(X'_i - X_i)\delta^{(2)}(\mathbf{k}'_{\perp i} - \mathbf{k}_{\perp i})\delta(\lambda'^2 - \lambda^2)\delta_{h'h}$$

Scalar di-quark model (Yukawa theory)

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}\phi\left(\partial^2 + \lambda^2\right)\phi + g\bar{\psi}\psi\phi$$

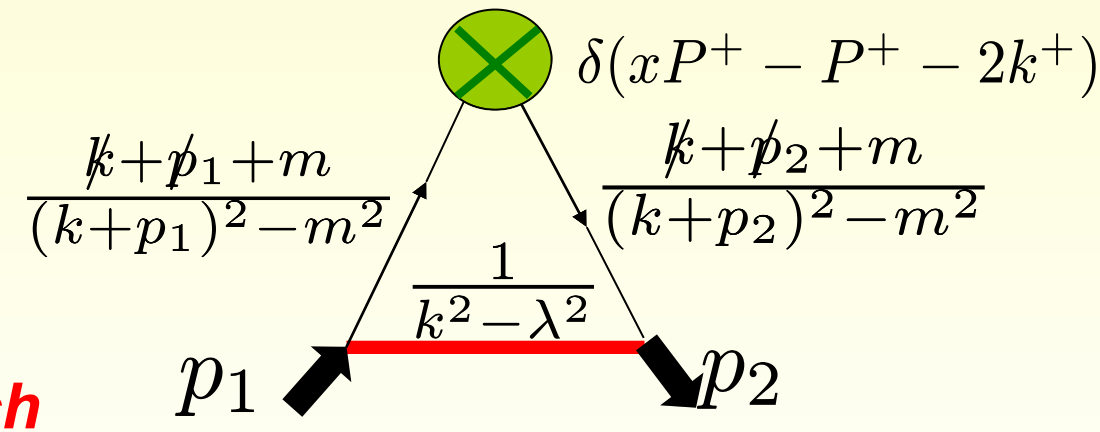
struck spin-*1/2* quark

collective scalar diquark spectator

coupling knows about spin

Diagrammatic approach:

via covariant time ordered perturbation theory



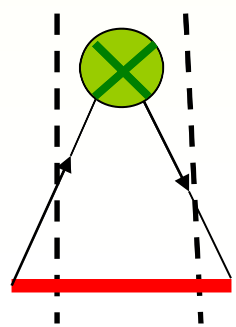
LC- Hamiltonian approach

$$k^\mu \rightarrow (k^+, k^-, \mathbf{k}_\perp),\ k^\pm = k^0 \pm k^3,\ \mathbf{k}_\perp = (k^1, k^2).$$

integrate out minus component to find LCWF

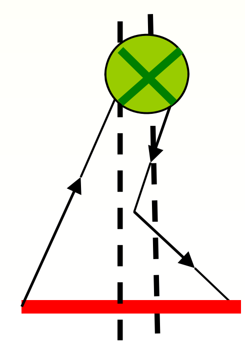
parton number conserved LCWF

(outer region)



parton number violating LCWF

(central region)



four parton number conserved LCWFs with angular momentum *L*

$$\psi^{\uparrow}_{+1/2}(X,\mathbf{k}_\perp) = \left(M+\frac{m}{X}\right)\varphi(x,\mathbf{k}_\perp)\,, \qquad L=0$$

$$\psi^{\downarrow}_{-1/2}(X,\mathbf{k}_\perp) = \left(M+\frac{m}{X}\right)\varphi(x,\mathbf{k}_\perp)\,, \qquad L=0$$

$$\psi^{\downarrow}_{+1/2}(X,\mathbf{k}_\perp) = \frac{k^1-ik^2}{X}\varphi(X,\mathbf{k}_\perp)\,, \qquad L=-1$$

$$\psi^{\uparrow}_{-1/2}(X,\mathbf{k}_\perp) = -\frac{k^1+ik^2}{X}\varphi(X,\mathbf{k}_\perp)\,, \qquad L=+1$$

in terms of one scalar LCWF (e.g., generalized Yukawa theory)

$$\varphi(X,\mathbf{k}_\perp) = \frac{gM^{2p}}{\sqrt{1-X}}X^{-p}\left(M^2-\frac{\mathbf{k}_\perp^2+m^2}{X}-\frac{\mathbf{k}_\perp^2+\lambda^2}{1-X}\right)^{-p-1}\,,$$

- Yukawa result follows with p=0
- p>0 serves as analytic regularization (model parameter)

axial-vector LCWFs have L= {0,+1,-1}

Unintegrated PDFs (diquark model)

- I view them as a phenomenological concept to describe data

? PDF counting, definition, factorization theorems, universality

✓ twist-2 and -3 PDFs are expressed by one LCWF overlap, e.g.

L=0 *L=1* *overlap (one should wonder)*

$$q(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2+\mathbf{k}_\perp^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)\,, \qquad \mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \int d\lambda^2 \rho(\lambda^2)\,\frac{\left|\phi(x,\mathbf{k}_\perp|\lambda^2)\right|^2}{1-x}$$

$$\Delta q(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2-\mathbf{k}_\perp^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)\,, \qquad \Delta_\perp q(x,\mathbf{k}_\perp^2) = \frac{k_1(m+xM)}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)$$

$$\delta q(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2-k_1^2+k_2^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2),\quad \delta_{\mathrm{L}} q(x,\mathbf{k}_\perp^2) = \frac{-2k_1(m+xM)}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)$$

✓ elimination of LCWF overlap yields 12-1 constraints

✓ known results **[Jakob et al. (97)]** are recovered with

$$\mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \frac{g^2}{M^2}\,\frac{(1-x)^{2p+1}}{\left[(1-x)\frac{m^2}{M^2}+x\frac{\lambda^2}{M^2}-x(1-x)+\frac{\mathbf{k}_\perp^2}{M^2}\right]^{2p+2}}$$

Field theoretical GPD definition

GPDs are defined as matrix elements of ***renormalized*** ***light-ray*** operators:

DM, Robaschik, Geyer, Dittes, Hořejši (94)

$$F(x,\eta,\Delta^2,\mu^2) = \int_{-\infty}^{\infty} d\kappa\, e^{i\kappa\, x\, n\cdot P} \langle P_2 | \mathcal{R}T : \phi(-\kappa n)[(-\kappa n),(\kappa n)]\phi(\kappa n): | P_1 \rangle,\ n^2 = 0$$

momentum fraction *x* , skewness $\eta = \frac{n\cdot\Delta}{n\cdot P}$ $\Delta = P_2 - P_1$ $P = P_1 + P_2$ $\Delta^2 \equiv t$

For a nucleon target we have four chiral even twist-two GPDs:

$$\bar\psi_i \gamma_+ \psi_i \quad \Rightarrow \quad {}^{i}q^{V} = \bar U(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

$$\bar\psi_i \gamma_+ \gamma_5 \psi_i \quad \Rightarrow \quad {}^{i}q^{A} = \bar U(P_2,S_2)\gamma_+\gamma_5 U(P_1,S_1) \widetilde{H}_i + \bar U(P_2,S_2)\frac{\gamma_5}{2M} U(P_1,S_1) \widetilde{E}_i$$

shorthands:

chiral even GPDs: $F = \{H, E, \widetilde{H}, \widetilde{E}\}$ & CFFs: $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

chiral odd GPDs: $F_T = \{H_T, E_T, \widetilde{H}_T, \widetilde{E}_T\}$ $\mathcal{F}_T = \{\mathcal{H}_T, \mathcal{E}_T, \widetilde{\mathcal{H}}_T, \widetilde{\mathcal{E}}_T\}$

GPD properties (from definition)

- polynomiality arises from Lorentz covariance
  (but GPDs are not Lorentz invariant or covariant)

$$\int_{-1}^{1} dx\, x^n F(x,\eta,t) = \text{polynom of order } n \text{ or } n+1 \text{ in } \eta$$

- satisfied within double distribution representation

$$F(x,\eta,t) = (1-x)^p \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-z\eta) f(y,z,t),\ \ p=\{0,1\}$$

- lowest moment: partonic form factor – related to observables
- first moment: expectation value of energy-momentum tensor **[Ji (96)]**
- reduction to parton densities (FPDs)

$$q(x) = \lim_{\Delta\to 0} H(x,\eta,t), \quad \Delta q(x) = \lim_{\Delta\to 0} \widetilde{H}(x,\eta,t)$$

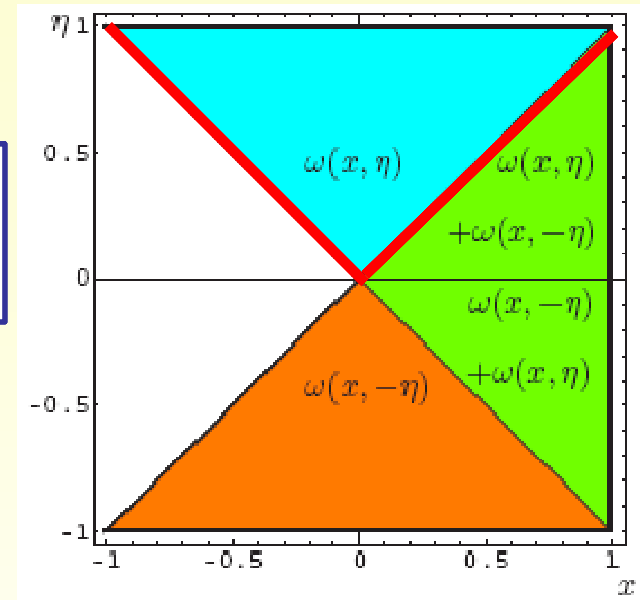
- positivity constraints (requirement on GPD and scheme) **[Pobylitsa(00,02)]**
  are automatically satisfied in the overlap representation

A partonic duality interpretation

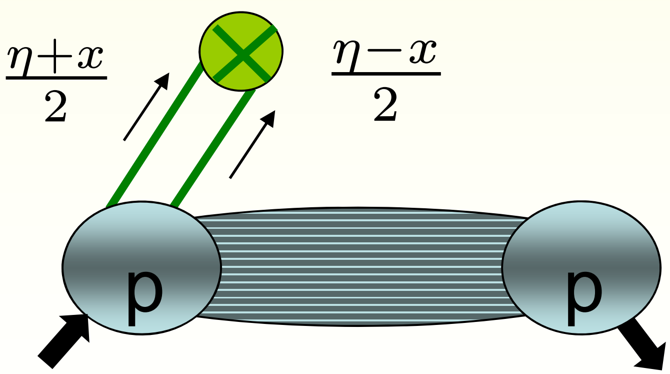
quark GPD (anti-quark x → -x):

$$F(x,\eta,t) \quad = \\ \theta(-\eta \le x \le 1)\,\omega(x,\eta,t) + \theta(\eta \le x \le 1)\,\omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f\,(y,(x-y)/\eta,t)$$

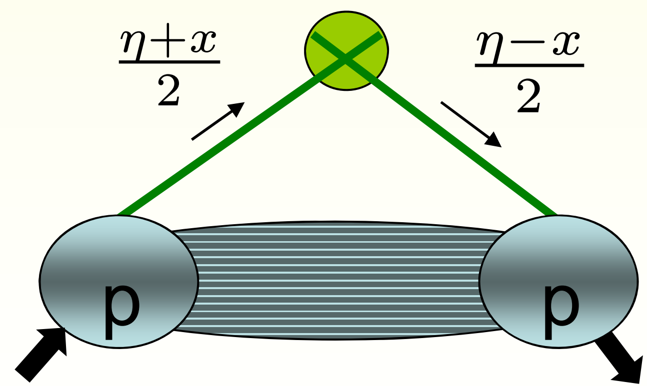


dual interpretation on partonic level:



support extension is unique **[DM et al. 92]**

ambiguous (D-term)
[DM, A. Schäfer (05), KMP-K (07)]



central region - η < x < η
mesonic exchange in t-channel

outer region η < x
partonic exchange in s-channel

Restoring full GPD: in the outer region GPDs are given as overlap of parton number conserved LCWFs, e.g.,

Hwang, DM (07)
Tiburzi, Miller (01,03)
Radyushkin et al. (03)

$$\frac{\Delta^1 - i\Delta^2}{2M} E(x \geq \eta, \eta, t))$$

$$= \sqrt{1-\zeta}\int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\uparrow *}_{+1/2}(X',\mathbf{k}'_\perp)\psi^{\downarrow}_{+1/2}(X,\mathbf{k}_\perp)+\psi^{\uparrow *}_{-1/2}(X',\mathbf{k}'_\perp)\psi^{\downarrow}_{-1/2}(X,\mathbf{k}_\perp)\right]$$

double distribution representation' for the outer region can be read off

$$E(x \geq \eta, \eta, t) = (1-x)\int_{\frac{x-\eta}{1-\eta}}^{\frac{x+\eta}{1+\eta}} \frac{dy}{\eta}\, e\left(y, \frac{x-y}{\eta}, t\right)$$

with double distribution

$$e(y,z,t) = N\frac{\left(\frac{m}{M}+y\right)\left((1-y)^2-z^2\right)^p}{\left[(1-y)\frac{m^2}{M^2}+y\frac{\lambda^2}{M^2}-y(1-y)-\left((1-y)^2-z^2\right)\frac{t}{4M^2}\right]^{2p+1}}$$

- central region follows by restoring the full support
- polynomiality is restored
- ambiguous modification of LCWFs is not allowed

Exploring the LCWF overlap

- ✓ duality between *s*- and *t*-channel allows to find the central region
- ➢ folklore that central region always restores polynomiality
  rather LCWFs have to respect hidden Lorentz covariance
- ✓ positivity constraints are automatically satisfied
- ✓ all twist-two and twist-three related quantities can be evaluated
- ✓ N, m, λ, and p can be used as model parameters
- ✓ concept: non-perturbative quantities in terms of LCWFs models

missing ingredients and open questions:

- Regge behavior as a collective phenomena is missing
- hard to understand from the naive point of view:

t and $k_\perp$ are tied to each other
μ^2 evolution arises from large $k_\perp$ behavior

Converting $\Delta_\perp$ into t dependence

unintegrated scalar LCWF overlap:

$$\mathbf{\Phi}(x \geq \eta, \eta, \mathbf{\Delta}_\perp, \mathbf{k}_\perp) = \frac{2-\zeta}{2\sqrt{1-\zeta}\sqrt{1-X'}\sqrt{1-X}} \phi^*\left(X', (\mathbf{k'}_\perp - \mathbf{\Delta'}_\perp)^2\right) \phi\left(X, \mathbf{k}_\perp^2\right)$$

$$\frac{1-X}{1-\zeta}\mathbf{\Delta}_\perp$$

has to possesses a scalar DD representation

$$\mathbf{\Phi}(x, \eta, t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x - y - z\eta)\, \hat{\mathbf{\Phi}}(y, z, t)$$

in terms of unintegrated double distribution

Mandelstam variable

$$\hat{\mathbf{\Phi}}(y, z, t) = \iint d\overline{\mathbf{k}}_\perp^2\, \hat{\mathbf{\Phi}}\left(y, z, t, \overline{\mathbf{k}}_\perp^2\right), \quad \overline{\mathbf{k}}_\perp = \mathbf{k}_\perp - (1 - y + z)\mathbf{\Delta}_\perp/2$$

the general ansatz (Laplace transform) solves the problem

$$\phi(X, \mathbf{k}_\perp) = \int_0^\infty d\alpha\, \varphi(X, \alpha) \exp\left\{-\alpha \frac{\mathbf{k}_\perp^2 - X(1-X)M^2}{(1-X)M^2}\right\}$$

Constrain for Laplace transformed LCWF

to restore polynomiality a (sufficient) constraint must be satisfied:

$$\frac{d}{d\eta}\left[\varphi^*\left(\frac{y-\eta(1-z)}{1-\eta}, A\frac{1-y+z}{2}\right)\varphi\left(\frac{y+\eta(1+z)}{1+\eta}, A\frac{1-y-z}{2}\right)\right]=0$$

a simple, however, non-trivial Laplace transformed LCWF:

$$\varphi(X,\alpha)=\varphi(\alpha)\exp\left\{-\alpha\frac{m^2}{M^2}-\alpha\frac{X}{1-X}\frac{\lambda^2}{M^2}\right\}$$

yields the desired result:

$$\varphi^*(X',\alpha_2)\varphi(X,\alpha_1)$$
$$\to\varphi^*\left(A\frac{1-y+z}{2}\right)\varphi\left(A\frac{1-y-z}{2}\right)\exp\left\{-A(1-y)\frac{m^2}{M^2}-Ay\frac{\lambda^2}{M^2}\right\}$$

NOTE: delta-like LCWF φ yields exponential like *t*-dependence
(disfavored at large *t*)

Spin structure of chiral even GPDs

specified spin coupling yields various GPD representations:

$$\left\{\begin{array}{c} H \\ E \\ \widetilde{H} \\ \widetilde{E} \end{array}\right\}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-\eta z)\left\{\begin{array}{c} h+(x-y)e \\ (1-x)e \\ \widetilde{h} \\ \widetilde{e}+(1-y-z/\eta)\,e \end{array}\right\}(y,z,t)$$

in terms of DDs:

$$\begin{aligned}
h &= \frac{(m+yM)^2}{M^2}\hat{\mathbf{\Phi}}(y,z,t) + \left[(1-y)^2-z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{M^2}\hat{\mathbf{\Phi}}(y,z,t')\right] \\
e &= 2\left(\frac{m}{M}+y\right)\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{h} &= \frac{(m+yM)^2}{M^2}\hat{\mathbf{\Phi}}(y,z,t) - \left[(1-y)^2-z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{M^2}\hat{\mathbf{\Phi}}(y,z,t')\right] \\
\widetilde{e} &= 2\left((1-y)^2-z^2\right)\hat{\mathbf{\Phi}}(y,z,t)
\end{aligned}$$

NOTE: $k_\perp$-integration can be replaced by t-integration

three constraints among four GPDs

Spin structure of chiral odd GPDs

chiral odd GPDs arise from the interference of *L=0* & *L=1* LCWFs

$$F_{\mathrm{T}}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, f_{\mathrm{T}}(y,z,t) \text{ for } F_{\mathrm{T}} = \left\{H_{\mathrm{T}}, E_{\mathrm{T}}, \widetilde{H}_{\mathrm{T}}, \widetilde{E}_{\mathrm{T}}\right\}$$

$$\begin{aligned}
h_{\mathrm{T}} &= \left[\left(\frac{m}{M}+y\right)^2 + \left((1-y)^2 - z^2\right)\frac{t}{4M^2}\right]\hat{\mathbf{\Phi}}(y,z,t) \\
e_{\mathrm{T}} &= 2\left[\left(\frac{m}{M}+y\right)(1-y) + (1-y)^2 - z^2\right]\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{h}_{\mathrm{T}} &= -\left[(1-y)^2 - z^2\right]\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{e}_{\mathrm{T}} &= 2\left(y+\frac{m}{M}\right) z\, \hat{\mathbf{\Phi}}(y,z,t)
\end{aligned}$$

(common DD representation)

obviously, another set of constraints, e.g.,

$$\begin{aligned}
h_{\mathrm{T}}(y,z,t) &= \frac{1}{2}\left[h+\widetilde{h}\right](y,z,t) + \frac{t}{8M^2}\widetilde{e}(y,z,t), \quad \widetilde{h}_{\mathrm{T}}(y,z,t) = -\frac{1}{2}\widetilde{e}(y,z,t) \\
e_{\mathrm{T}}(y,z,t) &= (1-y)e(y,z,t) + \widetilde{e}(y,z,t)\,, \quad \widetilde{e}_{\mathrm{T}}(y,z,t) = z\, e(y,z,t)
\end{aligned}$$

Regge improved PDFs and GPDs

collective degrees of freedom:

- *g* an effective coupling (normalization fixed by parton number)
- *m* struck quark mass (containing a bunch of partons)
- *λ* spectator diquark mass

[Landshoff, Polkinghorne (71) Pobylitsa (03)]

naively, any transformation within these parameters is allowed

$$\rho_\alpha(\lambda,\lambda_c) = \theta(\lambda-\lambda_c)\frac{2^{-\alpha}\Gamma(2+2p)}{\Gamma(2+2p-\alpha)\Gamma(\alpha)} M^{-2\alpha}(\lambda^2-\lambda_c^2)^{\alpha-1}$$

this simple Regge ansatz with intercept *α yields PDFs and DDs:*

$$\mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \frac{g^2}{M^2}\frac{x^{-\alpha}(1-x)^{2p+1+\alpha}}{\left[(1-x)\frac{m^2}{M^2}+x\frac{\lambda^2}{M^2}-x(1-x)+\frac{\mathbf{k}_\perp^2}{M^2}\right]^{2p+2}}$$

$$\hat{\mathbf{\Phi}}(y,z,t) = N\frac{y^{-\alpha}\left((1-y)^2-z^2\right)^{p+\alpha/2}}{\left[(1-y)\frac{m^2}{M^2}+y\frac{\lambda^2}{M^2}-y(1-y)-\left((1-y)^2-z^2\right)\frac{t}{4M^2}\right]^{2p+1}}$$

NOTES:

- a *t*-dependent *α(t)* might violate positivity constraints
- linear Regge trajectory has not been established from field theory
- *t*-dependence requires a more intricate integral transformation

- there is a mismatch of dimensional counting (P) and UV behavior
- Drell-Yan (perturbative behavior) and West (IR behavior) derived the same exclusive-inclusive (form factor-PDF) relation

- collinear gluon degrees of freedom are effectively absorbed
- twist-3 rel. unintegrated PDFs are affected by transverse gluons

- matching scale is ambiguous (evolution below $1\ GeV^2$???)

Model versus phenomenology

only for the scalar content of the proton, related to a ud-diquark
SU(6) symmetry states

$$|p\rangle = \frac{1}{\sqrt{2}}|u,(ud,0)^+\rangle + \frac{1}{\sqrt{6}}|u,(ud,0)^-\rangle - \frac{1}{\sqrt{3}}|d,(uu,1)^-\rangle$$

unintegrated PDFs (as below + Regge improved LCWF overlap)

GPDs:

$$\begin{aligned} H(x,\eta,t) &= \frac{1}{3}\left[2H_{u_{\rm val}}(x,\eta,t) - H_{d_{\rm val}}(x,\eta,t)\right], \\ E(x,\eta,t) &= \frac{1}{3}\left[2E_{u_{\rm val}}(x,\eta,t) - E_{d_{\rm val}}(x,\eta,t)\right], \end{aligned}$$

(generalized) form factors

$$F_1(t) = \int_{-\eta}^{1} dx H(x,\eta,t) \quad F_2(t) = \int_{-\eta}^{1} dx E(x,\eta,t)$$

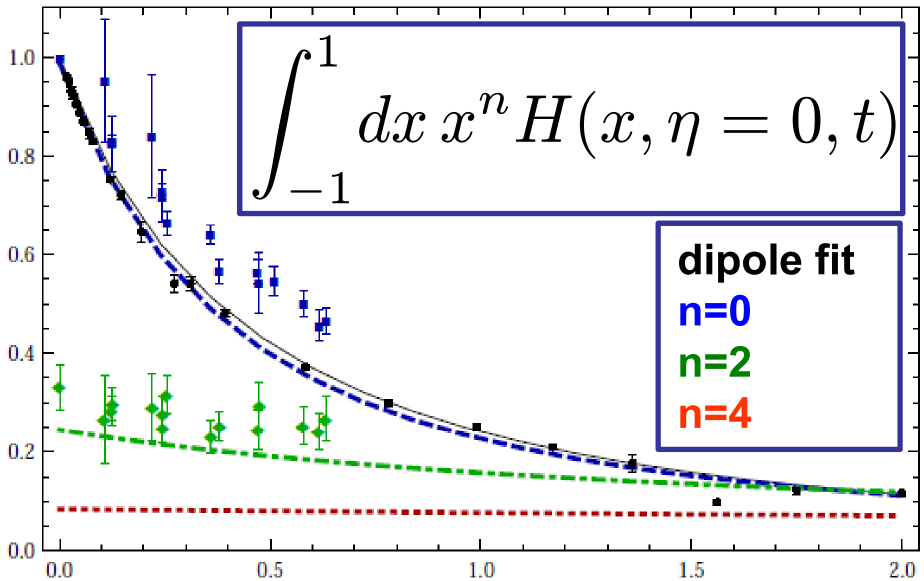
parton distribution functions

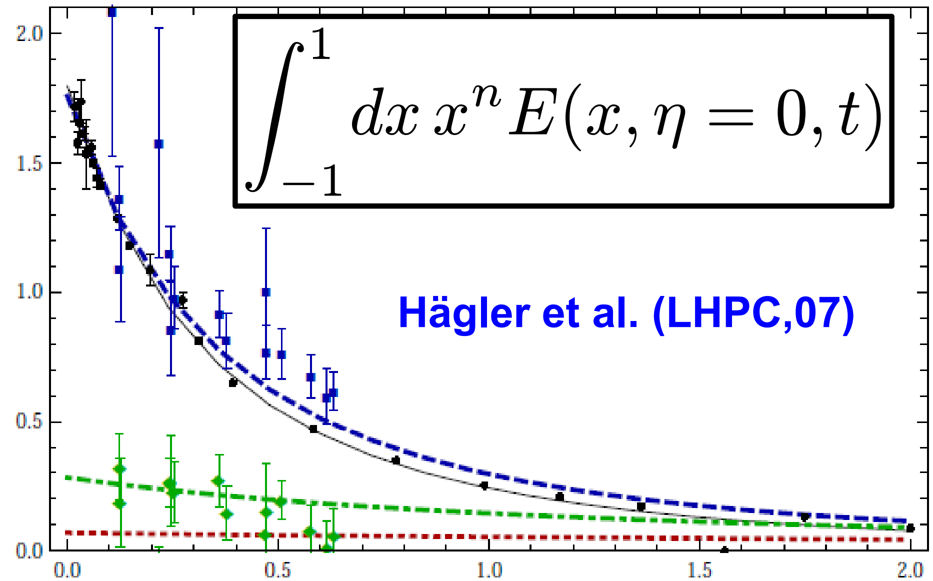
$$q(x) = H(x,\eta=0,t=0)$$

(generalized) form factors

$$\alpha = 0.45\,, \quad \alpha' = 0.9/\mathrm{GeV}^2\,, \quad p = 0.55\,, \quad \lambda = 0.87\,\mathrm{GeV}\,, \quad m = 0.45\,\mathrm{GeV}$$

$$\Rightarrow \quad R_{\mathrm{sca}} = 0.76\,\mathrm{fm}\,, \quad \kappa_{\mathrm{sca}} = 1.79$$





- *p < 1* (canonical value) and *0<α'*
- experimental and lattice data are well described:

✓ charge radius and magnetic moment

✓ *t*-dependence of (generalized) form factors

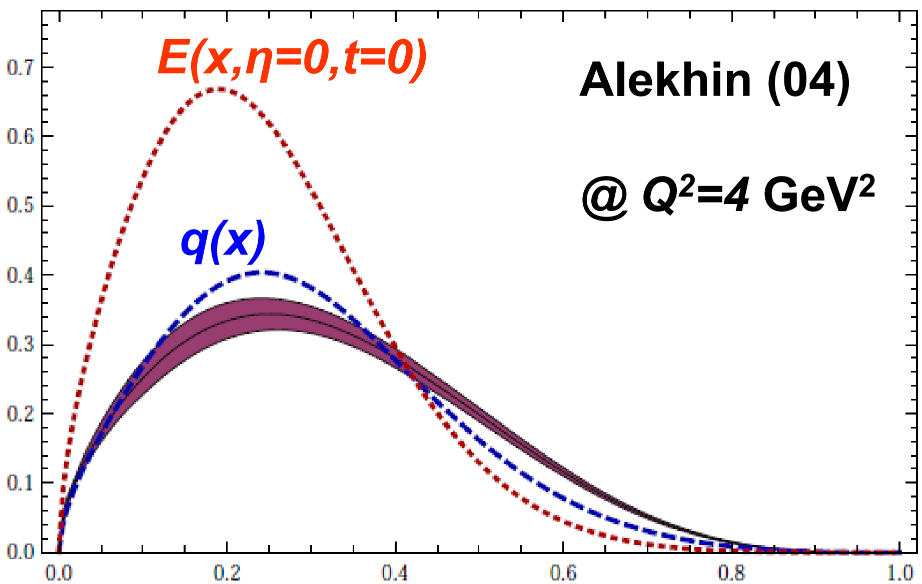
PDFs

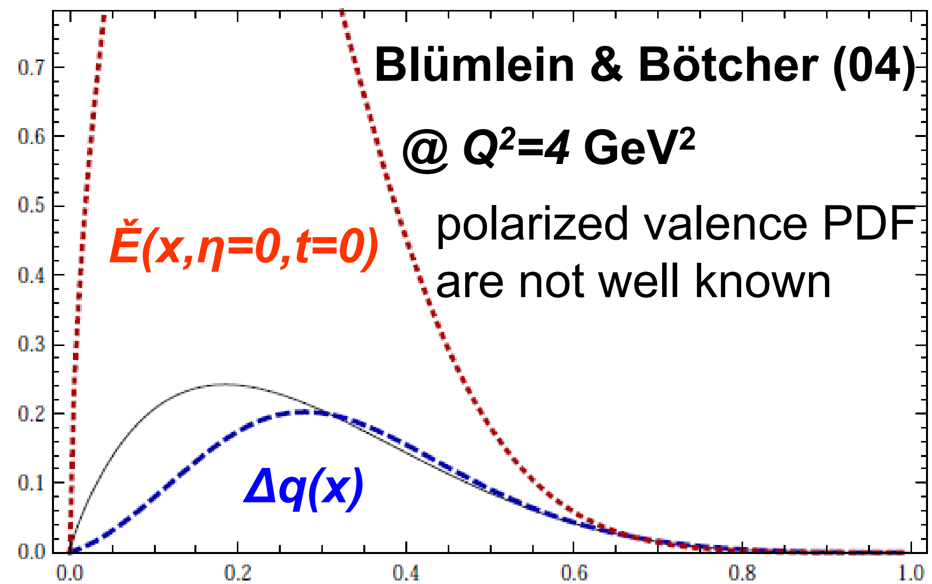
$$q(x) = \frac{(m+xM)^2 + \langle\langle \mathbf{k}_\perp^2 \rangle\rangle(x)}{M^2}\, \mathbf{\Phi}(x), \quad \Delta q(x) = \frac{(m+xM)^2 - \langle\langle \mathbf{k}_\perp^2 \rangle\rangle(x)}{M^2}\, \mathbf{\Phi}(x)$$

$$\mathbf{\Phi}(x) = \frac{\pi g^2}{2p+1} \frac{x^{-\alpha}(1-x)^{2p+1+\alpha}}{\left[(1-x)\frac{m^2}{M^2} + x\frac{\lambda^2}{M^2} - x(1-x)\right]^{2p+1}}$$

$$\langle\langle \mathbf{k}_\perp^2 \rangle\rangle(x) = \frac{1}{2p}\left[(1-x)m^2 + x\lambda^2 - x(1-x)M^2\right]$$

slightly depends on *x*



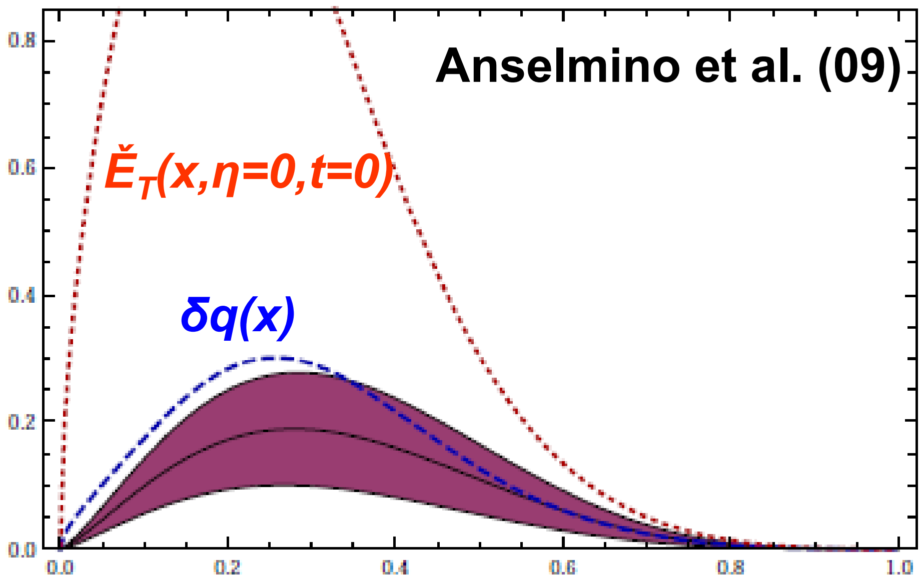


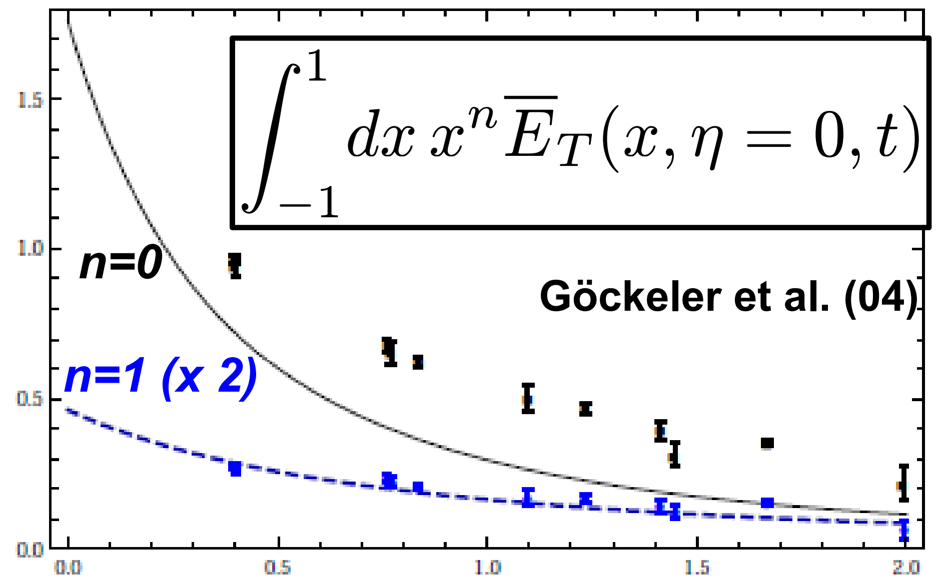
- ✓ reasonable agreement with phenomenological PDFs
- ✓ large target helicity non-conserved “PDFs”

Chiral odd PDFs (transversity)

- *L=0* and *L=1* overlap is maximal large transversity effects

$$\delta q(x,\mu^2) = \frac{1}{2}\left[q(x,\mu^2) + \Delta q(x,\mu^2)\right]$$ (Soffer bound is saturated)





✓ transversity is accessible in SIDIS [Anselmino et al.]

✓ chiral odd GPDs are seen in hard π^+ production [Goloskokov, Kroll 10]

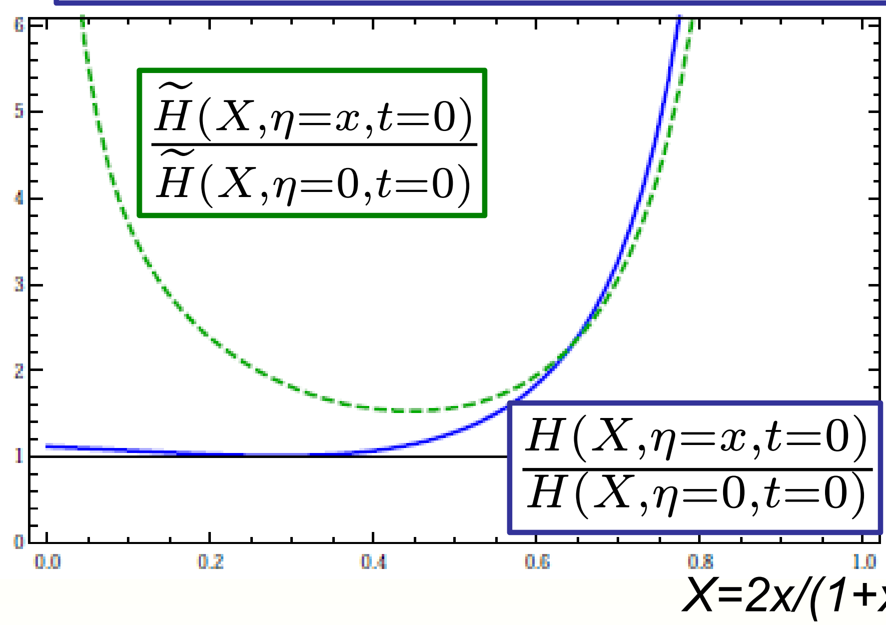
✓ some (almost no) indication in hard ρ^0 production (DVCS)

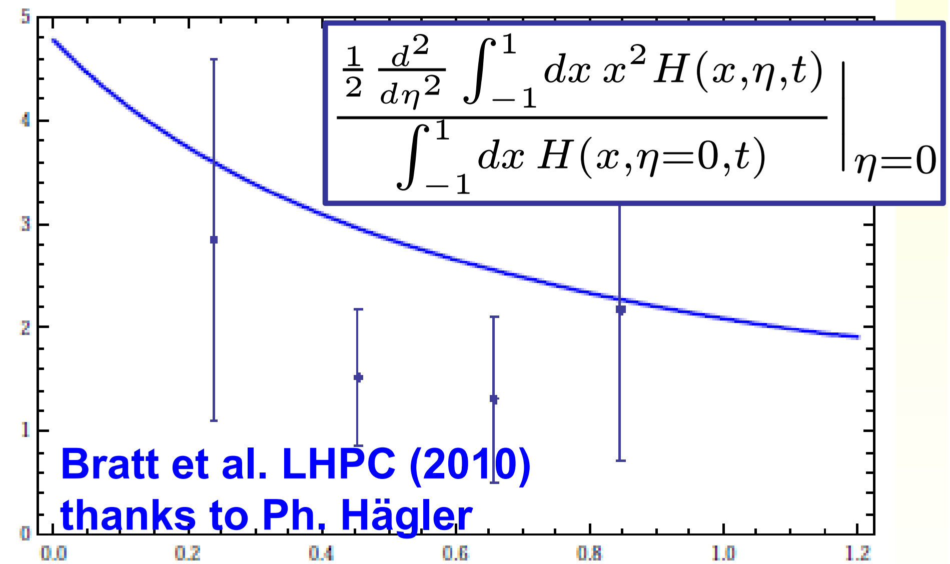
GPDs – skewness effect

for a LO description only the GPD on the cross-over line is needed

$$\mathcal{H}(\xi,t,\mathcal{Q}^2) \stackrel{\text{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right) H(x,\eta=\xi,t,\mathcal{Q}^2)$$

$$\stackrel{\text{LO}}{=} i\pi H^{-}(x=\xi,\eta=\xi,t,\mathcal{Q}^2) + \text{PV}\int_0^1 dx \frac{2x}{\xi^2 - x^2} H^{-}(x,\eta=\xi,t,\mathcal{Q}^2)$$



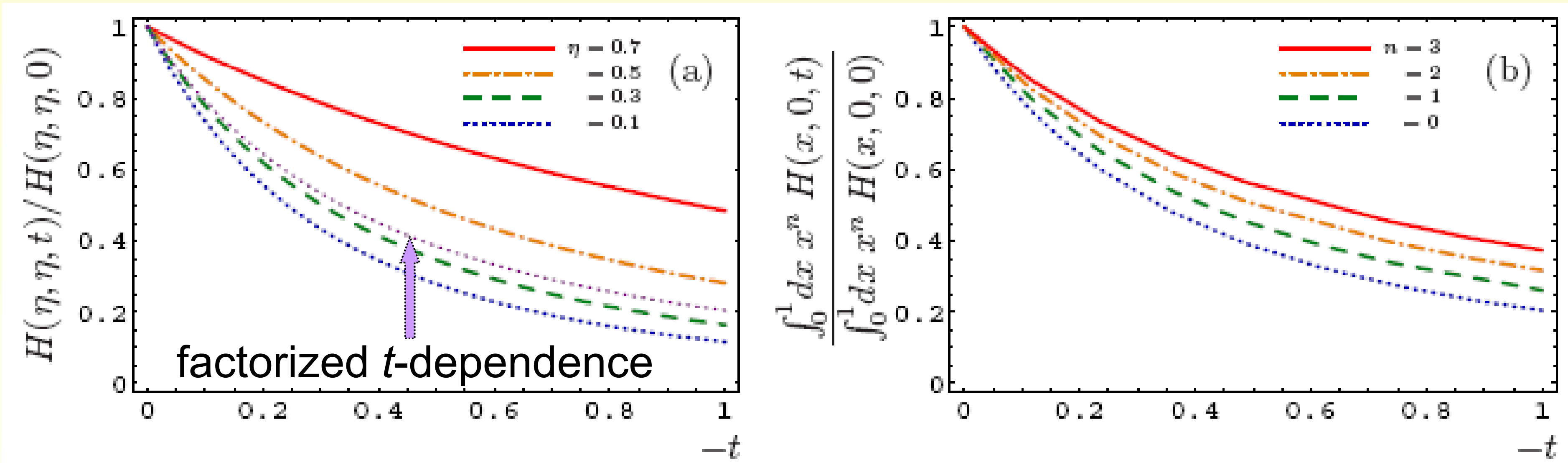


- ✓ *H*: small (big) skewness effect at small (large) x
- ? $\hat{H}$: large skewness effect
- ✓ large skewness effect in moments

GPDs: *t*-dependence

$H(\eta, \eta, t)$ on the cross-over line

Mellin moments at η=0



t- dependence gets

- flatter with increasing η
- steeper with decreasing η
- coincides with common picture

- qualitatively in agreement with lattice simulations

GPDs: so-called D-term

- ✓ thanks to spin-flip, i.e., *1-x* term outside of GPD *E*, polynomiality is completed
- ✓ duality is respected, no ambiguous *D*-term
- ✓ projecting on the *D*-term $$D(x,t) = -\lim_{\eta\to\infty} E(x\eta,\eta,t)$$

agreement with chiral quark soliton model: *0.33* **[Wakamatsu (07)]**

$$D(x,t=0) \approx (1-x^2)\left[0.345\, C_1^{3/2}(x) - 0.163\, C_3^{3/2}(x) + 0.026\, C_5^{3/2}(x) + \cdots\right]$$

- ✓ for "pomeron" exchange $\alpha \gtrsim 1$ we find a large negative *D*-term

[Polyakov et al (01)]

Conclusions

unintegrated PDFs and GPDs are given as LCWF overlap

- scalar spectator diquark model is well understood
- simple for (unintegrated) PDFs and intricate for GPDs
- Regge improvement from s-channel is to reconsider
- a simple model describes phenomenological findings

quark orbital angular momentum is an appropriate label:

- axial-vector part is so far restricted to forward case
- $|L| \leqq 1$ for scalar and axial-vector exchanges
- $|L| \geqq 2$ requires tensor (rank $\geqq 2$) exchanges
- angular momentum appears everywhere, e.g., in PDFs
- spin puzzle can be simply quantified in terms of LCWFs