TRANSVERSITY AND T-ODD DISTRIBUTIONS FROM DRELL-YAN PROCESSES AT COMPASS

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Subjects for investigation

Transversity: $h_1(x) \equiv \Delta_T q(x) \equiv \delta q(x)$

Boer-Mulders function: $h_1^{\perp}(x,\vec{k}_T^2) \equiv \Delta_T^0 f(x,\vec{k}_T^2) \equiv -\frac{M}{k_T}\Delta^N f_{q\uparrow/N}(x,\vec{k}_T^2)$

$$\Delta^N f_{q^\uparrow/p} = \hat{f}_{q^\uparrow/p} - \hat{f}_{q^\downarrow/p}$$

Sivers function: $f_{1T}^{\perp}(x,\vec{k}_T^2) \equiv \Delta_0^T f(x,\vec{k}_T^2) \equiv -\frac{M}{2k_T}\Delta^N f_{q/N^\uparrow}(x,\vec{k}_T^2)$

$$\Delta^N f_{q/p^\uparrow} = \hat{f}_{q/p^\uparrow} - \hat{f}_{q/p^\downarrow}$$

Collins function: $\Delta_T^0 D_{h/q} \equiv \frac{1}{2}\Delta^N D_{h/q^\uparrow}$

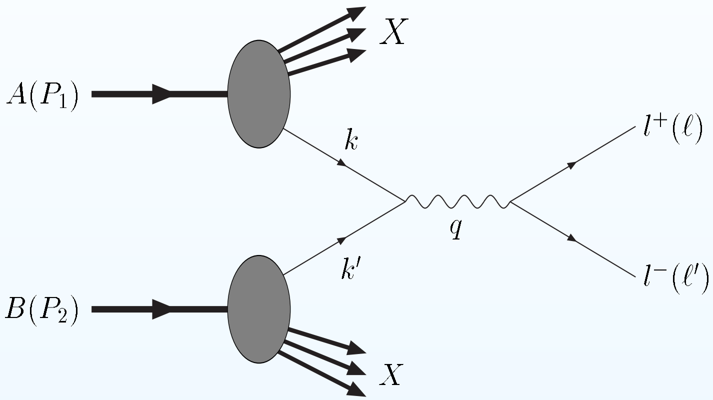
$$\Delta^N D_{h/q^\uparrow} = \hat{D}_{h/q^\uparrow} - \hat{D}_{h/q^\downarrow}$$

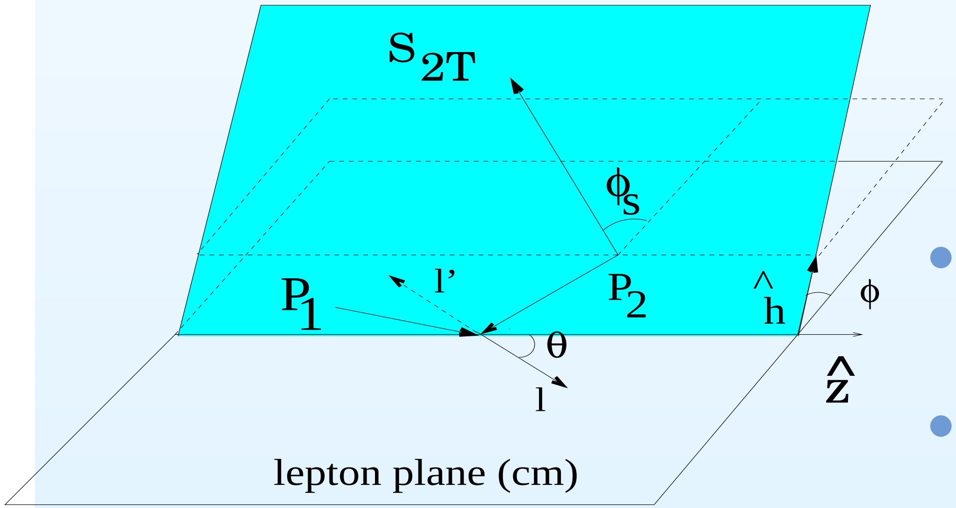
SIDIS:

$$A_{Coll}^{p,\pi^+} \simeq \frac{4\Delta_T u_V\ \Delta_T^0 D_1 + \Delta_T d_V\ \Delta_T^0 D_2}{4u_V D_1 + d_V D_2} \qquad A_{Coll}^{p,\pi^-} \simeq \frac{4\Delta_T u_V\ \Delta_T^0 D_2 + \Delta_T d_V\ \Delta_T^0 D_1}{4u_V D_2 + d_V D_1}$$

$$A_{Coll}^{d,\pi^+} \simeq \frac{(\Delta_T u_V + \Delta_T d_V)(4\Delta_T^0 D_1 + \Delta_T^0 D_2)}{(u_V + d_V)(4D_1 + D_2)} \qquad A_{Coll}^{d,\pi^-} \simeq \frac{(\Delta_T u_V + \Delta_T d_V)(\Delta_T^0 D_1 + 4\Delta_T^0 D_2)}{(u_V + d_V)(D_1 + 4D_2)}$$

Kinematics





- $x_1 = \frac{Q^2}{2p_1 q}$, $x_2 = \frac{Q^2}{2p_2 q}$ **– fractions of the longitudinal momentum of the hadrons A and B carried by the quark and antiquark which annihilate into virtual photon**
- $s = (p_1 + p_2)^2 \simeq 2p_1 p_2$ **– the center of mass energy squared**

  $Q^2 = M^2 \simeq x_1 x_2 s \equiv \tau s$

  $y = \frac{1}{2} \ln \frac{x_1}{x_2}$

  $x_f = x_1 - x_2$

  $x_1 = \frac{\sqrt{x_f^2 + 4\tau} + x_F}{2} = \sqrt{\tau} e^y$

  $x_2 = \frac{\sqrt{x_f^2 + 4\tau} - x_F}{2} = \sqrt{\tau} e^{-y}$
- θ **– production angle in the dilepton rest frame – polar angle of the lepton pair in the dilepton rest frame**
- ϕ **– azimuthal angle of lepton pair**
- ϕ_S **– azimuthal angle of the hadron polarization measured with respect to lepton plane**

Cross-sections

QPM: (D. Boer, PRD 60 (1999) 014012)

Unpolarized DY: $H_1 H_2 \longrightarrow l^+ l^- X$

$$\frac{d\sigma^{(0)}(H_1 H_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2} \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[\bar{f}_{1q} f_{1q}] +$$

$$\sin^2\theta \cos(2\phi) \mathcal{F}\left[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}) \frac{\bar{h}_{1q}^{\perp} h_{1q}^{\perp}}{M_1 M_2} \right] \Big\}$$

Single polarized DY: $H_1 H_2^{\uparrow} \longrightarrow l^+ l^- X$

PAX: $\bar{p} p^{\uparrow} \longrightarrow e^+ e^- X$

COMPASS: $\pi^- p^{\uparrow} \longrightarrow \mu^+ \mu^- X$

$$\frac{d\sigma^{(1)}(H_1 H_2^{\uparrow} \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2} \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[f_1 \bar{f}_1]$$

$$+\sin^2\theta \cos(2\phi) \mathcal{F}\left[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}) \frac{h_1^{\perp} \bar{h}_1^{\perp}}{M_1 M_2} \right]$$

$$+(1+\cos^2\theta)\sin(\phi - \phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{1T} \frac{f_{1T}^{\perp} \bar{f}_1}{M_1} \right] - \sin^2\theta \sin(\phi + \phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} \frac{h_1 \bar{h}_1^{\perp}}{M_2} \right] \Big\}$$

$$\hat{h} \equiv \mathbf{q}_T / |\mathbf{q}_T|$$

$$\mathcal{F}[f\bar{f}] \equiv \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \delta^2(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T) f_q(x_1, \mathbf{k}_{1T}^2) \bar{f}_q(x_2, \mathbf{k}_{2T}^2)$$

Unpolarized DY $H_1 H_2 \to l^+ l^- X$

$$\frac{d\sigma^{(0)}(H_1 H_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2}\sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[\bar{f}_{1q} f_{1q}] + \sin^2\theta\cos(2\phi)\mathcal{F}\Big[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\frac{\bar{h}_{1q}^{\perp} h_{1q}^{\perp}}{M_1 M_2}\Big]\Big\}$$

$$\begin{aligned} R &= \frac{3}{16\pi}(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi \\ &+ k\sin^2\theta\cos 2\phi), \quad (\lambda \simeq 1, \mu \simeq 0, k \equiv \nu/2) \end{aligned}$$

$$h_{1q}^{\perp}(x, \mathbf{p}_T^2) = \frac{\alpha_T}{\pi}\frac{M_C M_H}{\mathbf{p}_T^2 + M_C^2} \mathrm{e}^{-\alpha_T \mathbf{p}_T^2} f_{1q}(x) \qquad (\mathrm{M}_c = 2.3\,GeV, \alpha_T = 1GeV^{-2})$$

$\mathbf{q}_T$ integration approach

- e^+e^- **annihilation:** **D. Boer, R. Jakob, P.J. Mulders, NPB 504, 345 (1997); PLB 424, 143 (1998)**
- **SIDIS:** **A.M. Kotzinian, P.J. Mulders, Phys. Lett. B406 (1997) 373**
- **Single-polarized DY (Sivers function investigation) A. Efremov et al, Phys. Lett. B612 (2005)**

We introduce [Phys. Rev. D 72 (2005) 054027; Eur. Phys. Jorunal C, DOI: 10.1140/epjc/s2006-02490-1]

$$\hat{R} = \frac{\int d^2\mathbf{q}_T [|\mathbf{q}_T|^2/M_1M_2][d\sigma^{(0)}/d\Omega]}{\int d^2\mathbf{q}_T \sigma^{(0)}},$$

$$\hat{R} = \frac{3}{16\pi}(\gamma(1+\cos^2\theta) + \hat{k}\,\sin^2\theta\cos 2\phi)$$

Factorization

$$\hat{k} = \frac{\int d^2\mathbf{q}_T [\mathbf{q}_T^2/M_1 M_2] \sum_q e_q^2 \mathcal{F}[(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\frac{\bar{h}_1^\perp h_1^\perp}{M_1 M_2}]}{\int d^2\mathbf{q}_T \sum_q e_q^2 \mathcal{F}[\bar{f}_1 f_1]}$$

$$\mathcal{F}[f\bar{f}] \equiv \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \delta^2(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T) f_q(x_1,\mathbf{k}_{1T}^2)\bar{f}_q(x_2,\mathbf{k}_{2T}^2)$$

$$\delta^2(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)(2\frac{(\mathbf{q}_T\mathbf{k}_{1T})(\mathbf{q}_T\mathbf{k}_{2T})}{\mathbf{q}_T^2} - \mathbf{k}_{1T}\mathbf{k}_{2T})\mathbf{q}_T^2$$

$$= 2\,\mathbf{k}_{1T}^2\mathbf{k}_{2T}^2 + \underbrace{\mathbf{k}_{1T}^2(\mathbf{k}_{1T}\mathbf{k}_{2T})}_{\downarrow\ 0} + \underbrace{\mathbf{k}_{2T}^2(\mathbf{k}_{1T}\mathbf{k}_{2T})}_{\downarrow\ 0} + 2(\mathbf{k}_{1T}\mathbf{k}_{2T})^2 - 2(\mathbf{k}_{1T}\mathbf{k}_{2T})^2$$

$$\hat{k} = 8\frac{\sum_q e_q^2(\bar{h}_{1q}^{\perp(1)}(x_1)h_{1q}^{\perp(1)}(x_2)+(1\leftrightarrow 2))}{\sum_q e_q^2(\bar{f}_{1q}(x_1)f_{1q}(x_2)+(1\leftrightarrow 2))}$$

$$h_{1q}^{\perp(n)}(x) \equiv \int d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M^2}\right)^n h_{1q}^\perp(x,\mathbf{k}_T^2)$$

Single polarized DY process $H_1 H_2^{\uparrow} \to l^+ l^- X$

$$\frac{d\sigma^{(1)}(H_1 H_2^{\uparrow}\to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{12Q^2}\sum_q e_q^2 \Big\{ (1+\cos^2\theta)\mathcal{F}[f_1\bar{f}_1]$$

$$+\sin^2\theta\cos(2\phi)\mathcal{F}\left[\left(2\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\,\hat{\mathbf{h}}\cdot\mathbf{k}_{2T} - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}\right)\frac{h_1^{\perp}\bar{h}_1^{\perp}}{M_1 M_2}\right]$$

$$+(1+\cos^2\theta)\sin(\phi-\phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}\frac{f_{1T}^{\perp}\bar{f}_1}{M_1}\right]$$

$$-\sin^2\theta\sin(\phi+\phi_{S_1})\mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{k}_{2T}\frac{h_1\bar{h}_1^{\perp}}{M_2}\right]\Big\}$$

Let us consider SSA

$$\hat{A}_{h(f)} = \frac{\int d\Omega d\phi_{S_2}\sin(\phi\pm\phi_{S_2})[d\sigma(\mathbf{S}_{2T})-d\sigma(-\mathbf{S}_{2T})]}{\int d\Omega d\phi_{S_2}[d\sigma(\mathbf{S}_{2T})+d\sigma(-\mathbf{S}_{2T})]}$$

$$A_h = -\frac{1}{4}\frac{\sum_q e_q^2\,\mathcal{F}\left[\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_{1T}}{M_1}\bar{h}_{1q}^{\perp}h_{1q}\right]}{\sum_q e_q^2\,\mathcal{F}\left[\bar{f}_{1q}f_{1q}\right]},\quad A_f = \frac{1}{2}\frac{\sum_q e_q^2\,\mathcal{F}\left[\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_{2T}}{M_2}\bar{f}_1^q f_{1T}^{\perp q}\right]}{\sum_q e_q^2\,\mathcal{F}\left[\bar{f}_{1q}f_{1q}\right]}$$

$\downarrow$ $\downarrow$

A. Sissakian et al, PRD,2005

Anselmino et al, PRD, 2003; Efremov et al, PLB, 2005

Factorization

By analogy with $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ considered in ref. A. Efremov et al, PLB **612** (2005) 233, we introduce

$$\hat{A}_h = \frac{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T (|\mathbf{q}_T|/M_1) \sin(\phi+\phi_{S_2}) [d\sigma(\mathbf{S}_{2T}) - d\sigma(-\mathbf{S}_{2T})]}{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T [d\sigma(\mathbf{S}_{2T}) + d\sigma(-\mathbf{S}_{2T})]}$$

so that after integration

$$\hat{A}_h = -\frac{1}{2} \frac{\sum_q e_q^2 [\bar{h}_{1q}^{\perp(1)}(x_1) h_{1q}(x_2) + (1 \leftrightarrow 2)]}{\sum_q e_q^2 [\bar{f}_{1q}(x_1) f_{1q}(x_2) + (1 \leftrightarrow 2)]},$$

$$h_{1q}^{\perp(1)}(x) \equiv \int d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M^2}\right) h_{1q}^{\perp}(x, \mathbf{k}_T^2)$$

$\bar{p}p \to l^+l^-X$ **and** $\bar{p}p^\uparrow \to l^+l^-X$

By virtue of charge conjugation symmetry:

$$\hat{k}|_{\bar{p}p\to l^+l^-X} = 8\frac{\sum_q e_q^2[h_{1q}^{\perp(1)}(x_1)h_{1q}^{\perp(1)}(x_2)+\bar{h}_{1q}^{\perp(1)}(x_1)\bar{h}_{1q}^{\perp(1)}(x_2)]}{\sum_q e_q^2[f_{1q}(x_1)f_{1q}(x_2)+\bar{f}_{1q}(x_1)\bar{f}_{1q}(x_2)]},$$

$$\hat{A}_h|_{\bar{p}p^\uparrow\to l^+l^-X} = -\frac{1}{2}\frac{\sum_q e_q^2[h_{1q}^{\perp(1)}(x_1)h_{1q}(x_2)+\bar{h}_{1q}(x_1)\bar{h}_{1q}^{\perp(1)}(x_2)]}{\sum_q e_q^2[f_{1q}(x_1)f_{1q}(x_2)+\bar{f}_{1q}(x_1)\bar{f}_{1q}(x_2)]},$$

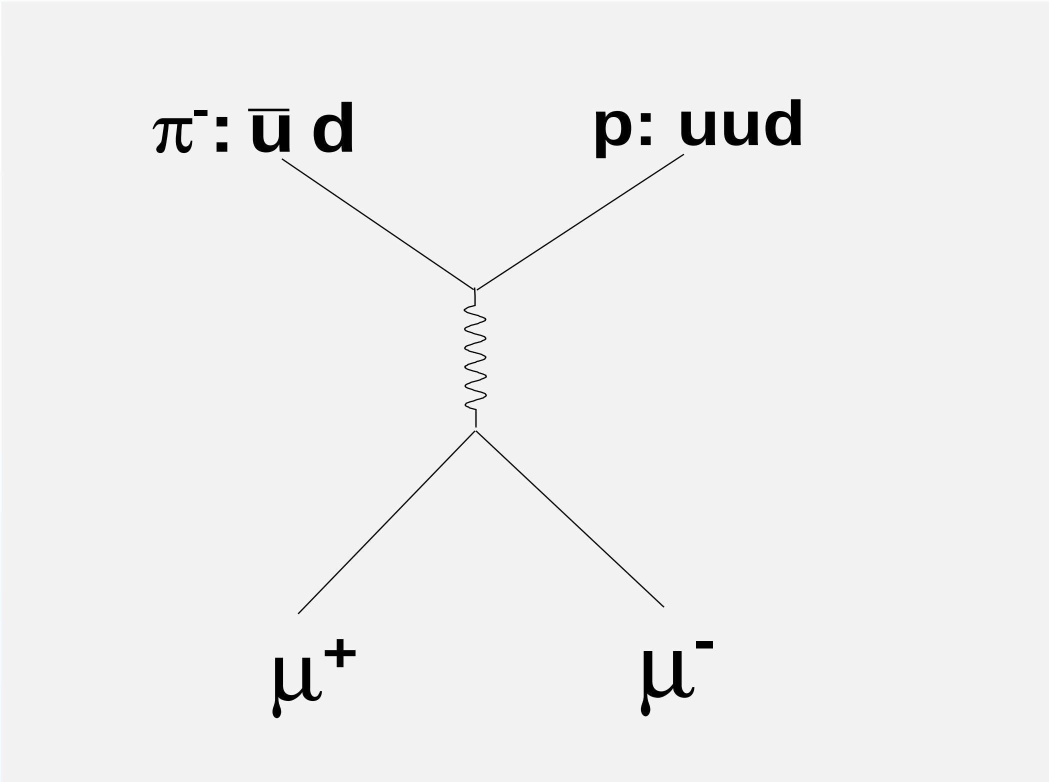
where now all PDF *refer to protons*. Neglecting squared antiquark and strange quark PDF contributions to proton and taking into account the quark charges and u quark dominance at large x, we get

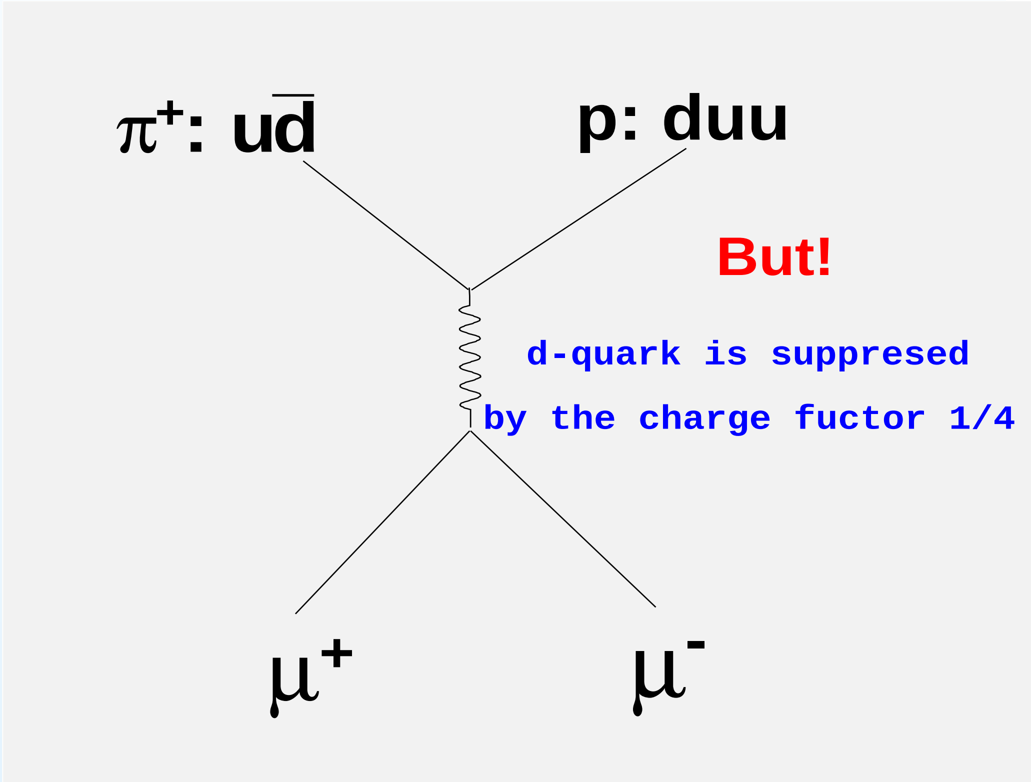
$$\hat{k}(x_1,x_2)|_{\bar{p}p\to l^+l^-X} \simeq 8\frac{h_{1u}^{\perp(1)}(x_1)h_{1u}^{\perp(1)}(x_2)}{f_{1u}(x_1)f_{1u}(x_2)},$$

$$\hat{A}_h(x_1,x_2)|_{\bar{p}p^\uparrow\to l^+l^-X} \simeq -\frac{1}{2}\frac{h_{1u}^{\perp(1)}(x_1)h_{1u}(x_2)}{f_{1u}(x_1)f_{1u}(x_2)}.$$

$\pi^- p \to \mu^+\mu^- X$ and $\pi^- p^\uparrow \to \mu^+\mu^- X$

Why $\pi^- p$ but not $\pi^+ p$?





$$\hat{k}(x_\pi, x_p)_{\pi^- p} \simeq 8 \frac{\bar{h}_{1u}^{\perp(1)}(x_\pi)\Big|_{\pi^-} h_{1u}^{\perp(1)}(x_p)\Big|_p}{\bar{f}_{1u}(x_\pi)\Big|_{\pi^-} f_{1u}(x_p)\Big|_p},$$

$$\hat{A}_h(x_\pi, x_p)_{\pi^- p^\uparrow} \simeq -\frac{1}{2} \frac{\bar{h}_{1u}^{\perp(1)}(x_\pi)\Big|_{\pi^-} h_{1u}(x_p)\Big|_p}{\bar{f}_{1u}(x_\pi)_{\pi^-} f_{1u}(x_p)_p}.$$

Assumption:

$$\frac{\bar{h}_{1u}^{\perp(1)}(x)_{\pi^-}}{h_{1u}^{\perp(1)}(x)_p} = C_u \frac{\bar{f}_{1u}(x)_{\pi^-}}{f_{1u}(x)_p}.$$

is consistent with the Boer's model, where

$$C_u = M_p c_\pi^u / M_\pi c_p^u$$

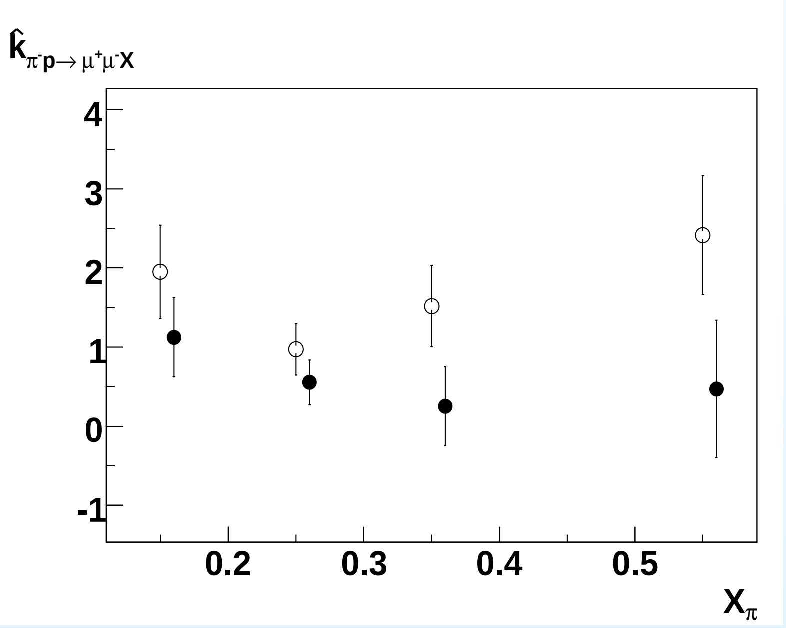
The simulations results show that $C_u \simeq 1$

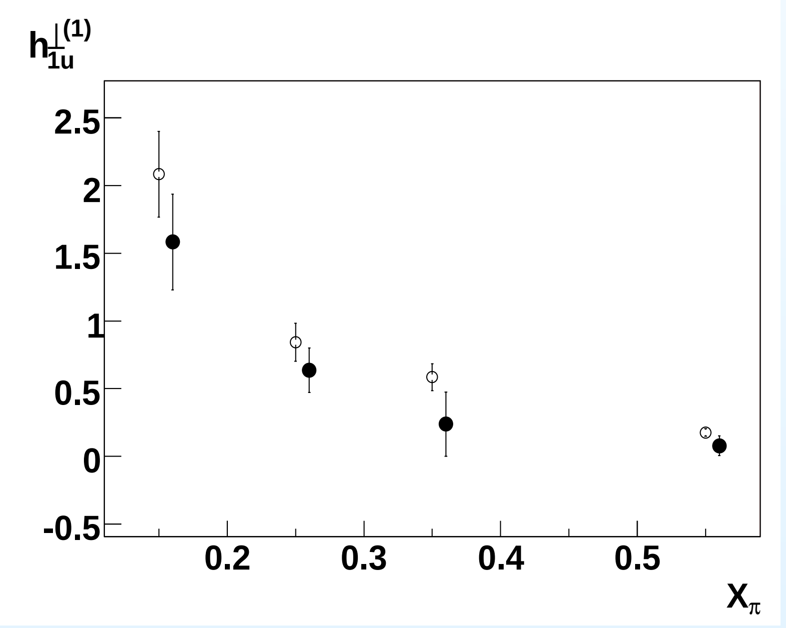
Simulation results (unpolarized DY)

$$x \equiv x_1 \simeq x_2 \ (x_F \simeq 0 \pm 0.4)$$

$$h_{1u}^{\perp(1)}(x) = f_{1u}(x)\sqrt{\frac{\hat{k}(x,x)}{8}}$$

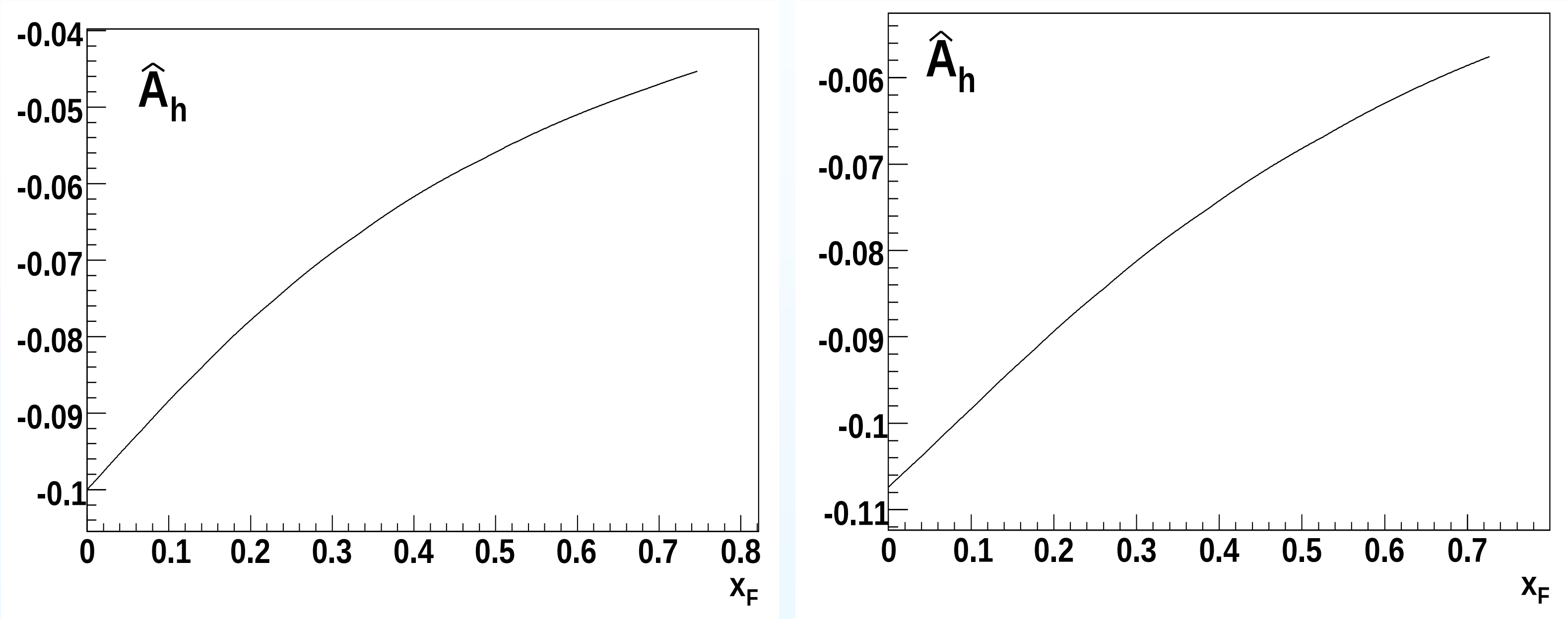
$$h_{1u(x)} = -4\sqrt{2}\frac{\hat{A}_h(x,x)}{\sqrt{\hat{k}(x,x)}}f_{1u}(x)$$





closed circles: 60 GeV, open cirles: 100 GeV

Estimation of SSA $\hat{A}_h$ (sigle-polarized DY)



Left: 100 GeV ($Q^2_{average} = 6.2\, GeV^2$), Right: 60 GeV ($Q^2_{average} = 5.5\, GeV^2$)

Feasibility study of introduced $\hat{k}$ and $\hat{A}_h$ and, therefore, of PDFs h_1 and $h_1^{\perp(1)}$ is now in progress (Torino-Dubna group + T. Iwata – see the talk at Workshop on Hadron Structure at J-PARC; can be downloaded from COMPASS talks page)

Sivers function from the single-polarized Drell-Yan

A. Efremov et al (Phys. Lett. B612 (2005))

q_T-integrated asymmetry

$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} = \frac{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T (|\mathbf{q}_T|/M_1) \sin(\phi-\phi_{S_2})[d\sigma(\mathbf{S}_{2T}) - d\sigma(-\mathbf{S}_{2T}}{\int d\Omega d\phi_{S_2} \int d^2\mathbf{q}_T [d\sigma(\mathbf{S}_{2T}) + d\sigma(-\mathbf{S}_{2T})]}$$

As a result:

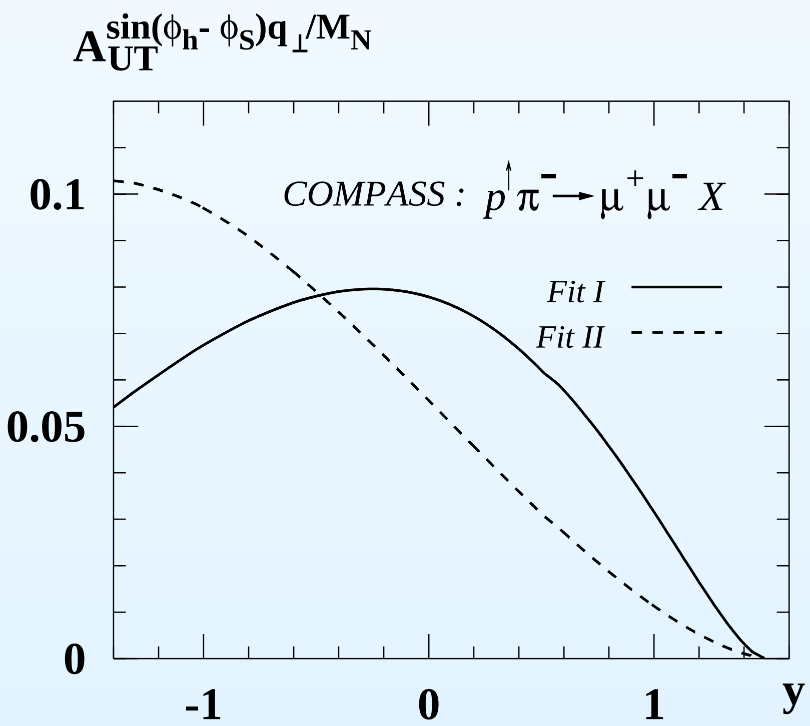
$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} = 2\,\frac{\sum_a e_q^2\, f_{1T}^{\perp(1)q/p}(x_1)\, f_1^{\bar{q}/\pi^-}(x_2)}{\sum_a e_q^2\, f_1^{q/p}(x_1)\, f_1^{\bar{q}/\pi^-}(x_2)},$$

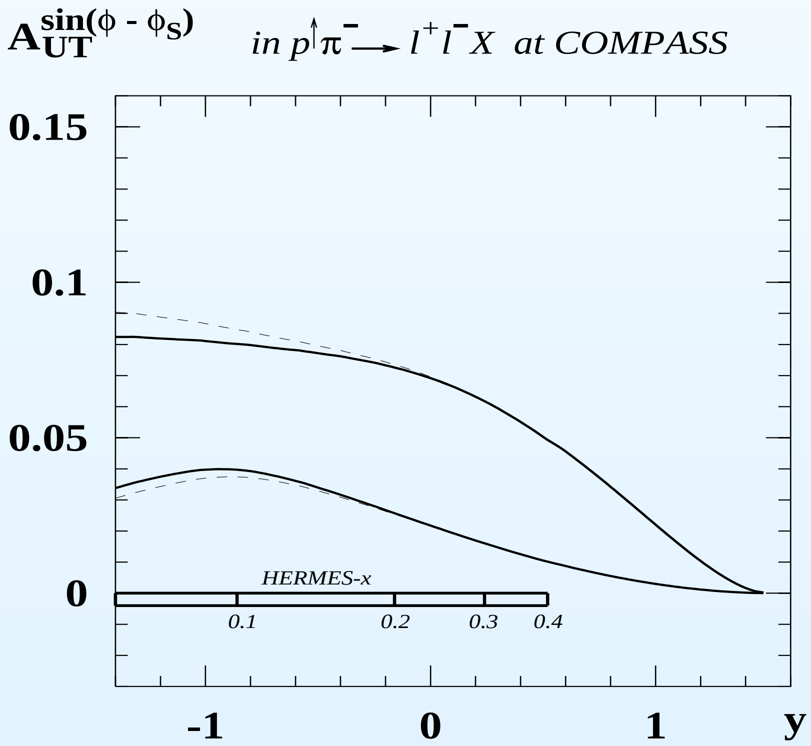
u-quark dominance for $\pi^- p$ DY:

$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} \simeq 2\,\frac{f_{1T}^{\perp(1)u/p}(x_1)\, f_1^{\bar{u}/\pi^-}(x_2)}{f_1^{u/p}(x_1)\, f_1^{\bar{u}/\pi^-}(x_2)},$$

Estimation of SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$

Two fits for Sivers function extracted from the HERMES data are used:
A. Efremov et al, Phys. Lett. B612 (2005)
J. Collins et al, hep-ph/0511272





Summary

DY $\pi^- p \to \mu^+ \mu^- X$ at COMPASS can provide us by:

- Purely unpolarized DY $\Rightarrow$ first moment of the Boer-Mulders function
- Single-polarized DY $\Rightarrow$ first moment of the Sivers function
- Both unpolarized and single-polarized DY $\Rightarrow$ transversity

Feasibility study of DY at COMPASS is now in progress (Torino-Dubna group + T. Iwata – see the talk by T. Iwata at Workshop on Hadron Structure at J-PARC, "Possibility of direct extraction of the transversity from polarized Drell-Yan measurement in COMPASS")

Simulations (testing)

Experiments on unpolarized DY: J.S. Conway et al, PRD **71** (2005) 074014; NA10 Collaboration, Z. Phys. C 31 (1986) 513, Z. Phys. C **37** (1988) 545

$$R = \frac{3}{16\pi}(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + k\sin^2\theta\cos 2\phi),$$
$$(\lambda \simeq 1, \mu \simeq 0, k \equiv \nu/2)$$