DRELL-YAN IN MODERATE ENERGY/MASS REGIME

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Walhalla Drell-Yan:

$Q = 50$ GeV, $Q_T << Q$.

Midgard Drell-Yan:

moderate photon masses ($Q = 2\text{-}7$ GeV/c^2),
medium size transverse momentum Q_T up to $Q/2$,
rest target, beam energy < 400 GeV

Drell-Yan is an ideal tool for studying a lot of things, at a condition:

parton model interpretation is a large part of the story.

Main question here:

at which extent we know what we see in a Midgard Drell-Yan?

Cross section $\approx$ $G_1(x_1,k_{T1})\ G_2(x_2,k_{T2})\ H(x_1,x_2,k_{T1},k_{T2})$

G(..) parton distribution functions

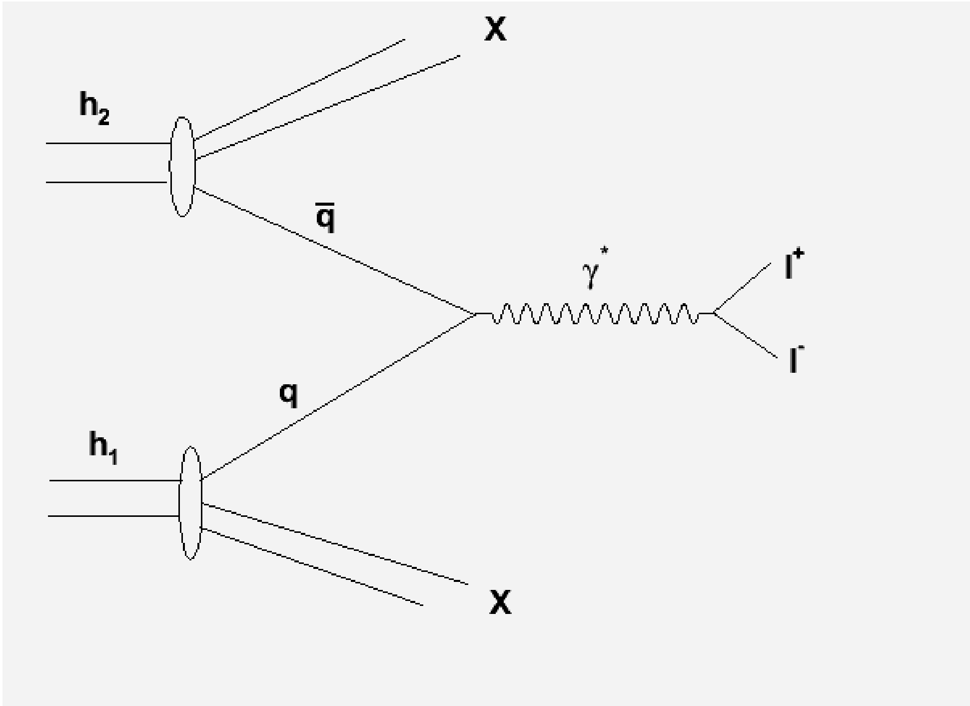
H(....) hard "parton-parton to dilepton plus something" cross section

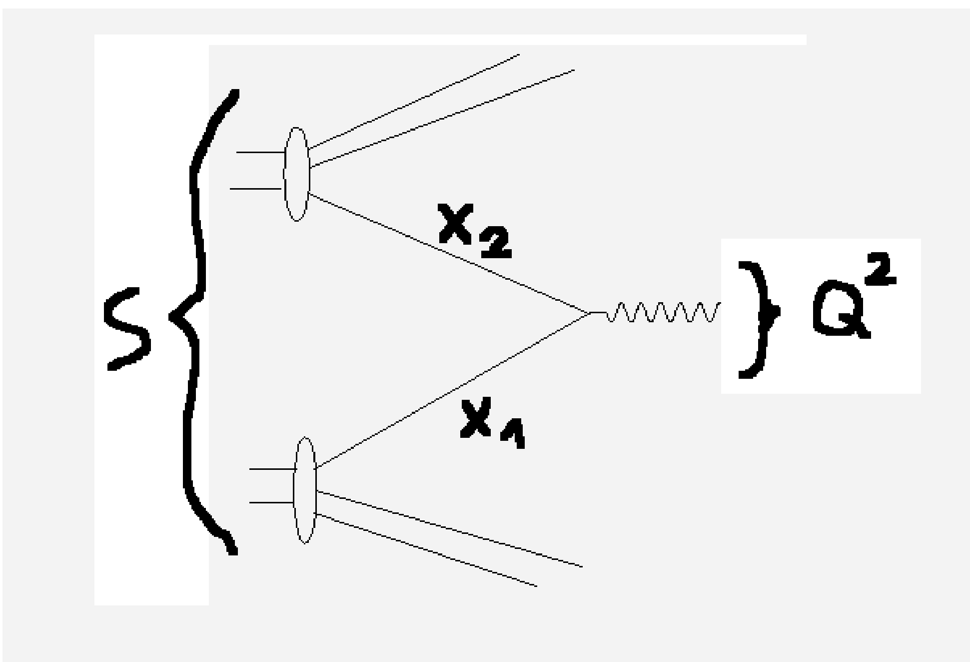
Two levels of analysis:

collinear analysis:
  parton collinear to parent hadron: distribution G(x)
  first order QCD corrections introduce transverse momentum via H(....)

TMD analysis
  parton has small k_T < 500 GeV/c already in $G(x,k_T)$

At parton model level,
  if $G(x,k_T) = G(x)G'(k_T)$ (frequent) collinear and transverse degrees of freedom are separately treated





S = total CM squared energy

Q2 = dilepton, virtual photon squared CM energy, or simply “mass”

$Q^2 = x_1\ x_2\ S$

$x_1\ x_2 \equiv \tau$ is the cm-energy transfer factor

Parton model: all the kinematics of the quark-antiquark pair is transferred to the dilepton

$\sigma = F(S,Q^2)\, G(x_1)\overline{G}(x_2)$ $\leftarrow$ sum over q, $\overline{q}$

where F includes S-dependent flux/phase space factor,
photon propagator $1/Q^4$,
two constant e.m. couplings of γ with quarks and leptons

So, at parton model level

(1) we have direct access to $G(x)$, $\overline{G}(x)$

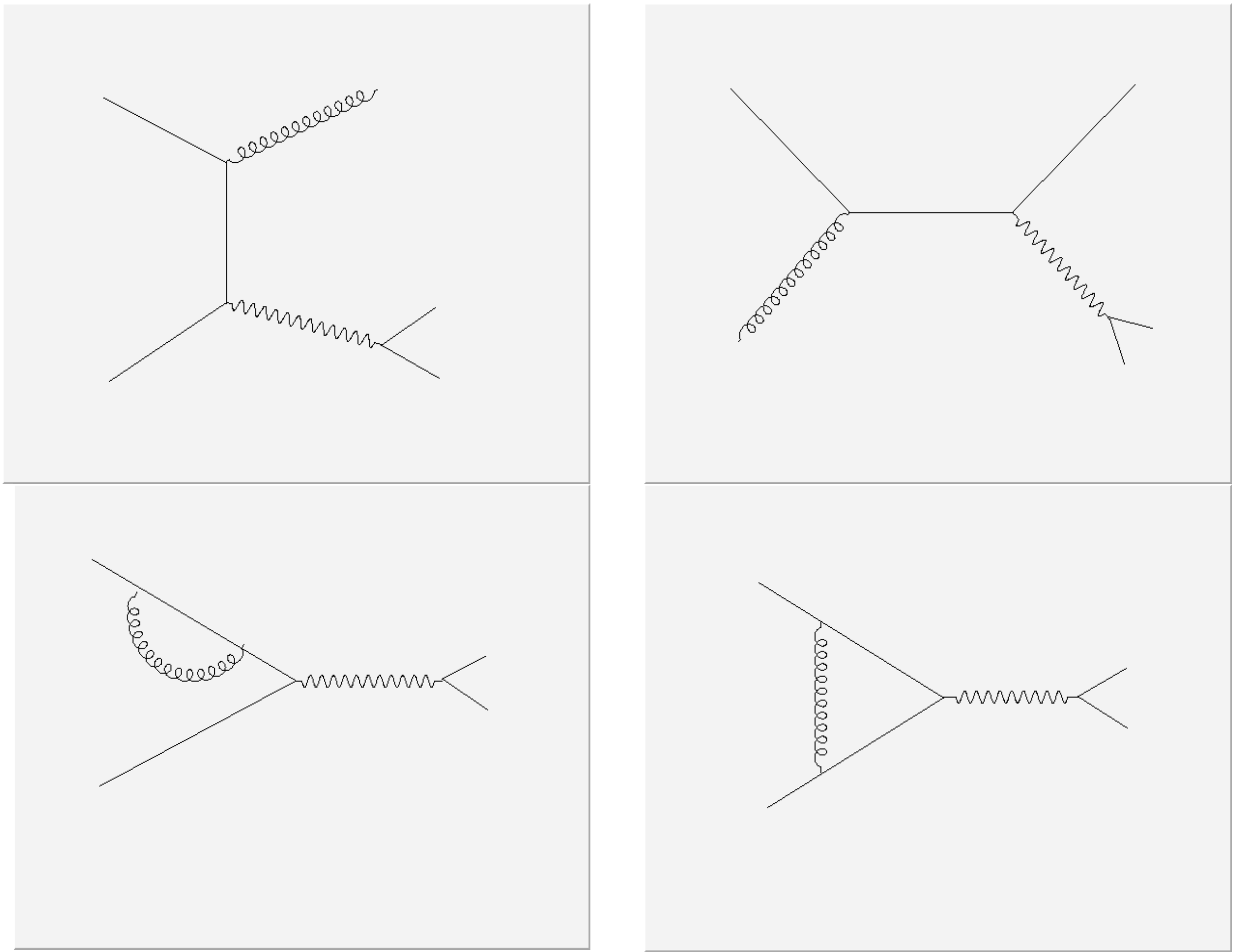
(2) scaling: several quantities may be defined
that only depend on x_1, x_2

Wonderful, but... believable or not? difficult answer

first order QCD corrections double $\frac{d\sigma}{dQ^2}$!

This is confirmed exp, for p, $\overline{p}$ Drell-Yan on nuclei.

First order PQCD corrections, real and virtual



When each diagram is multiplied by a complex conjugate, real graphs are multiplied by the specular graph, virtual graphs by the plain parton model graph.

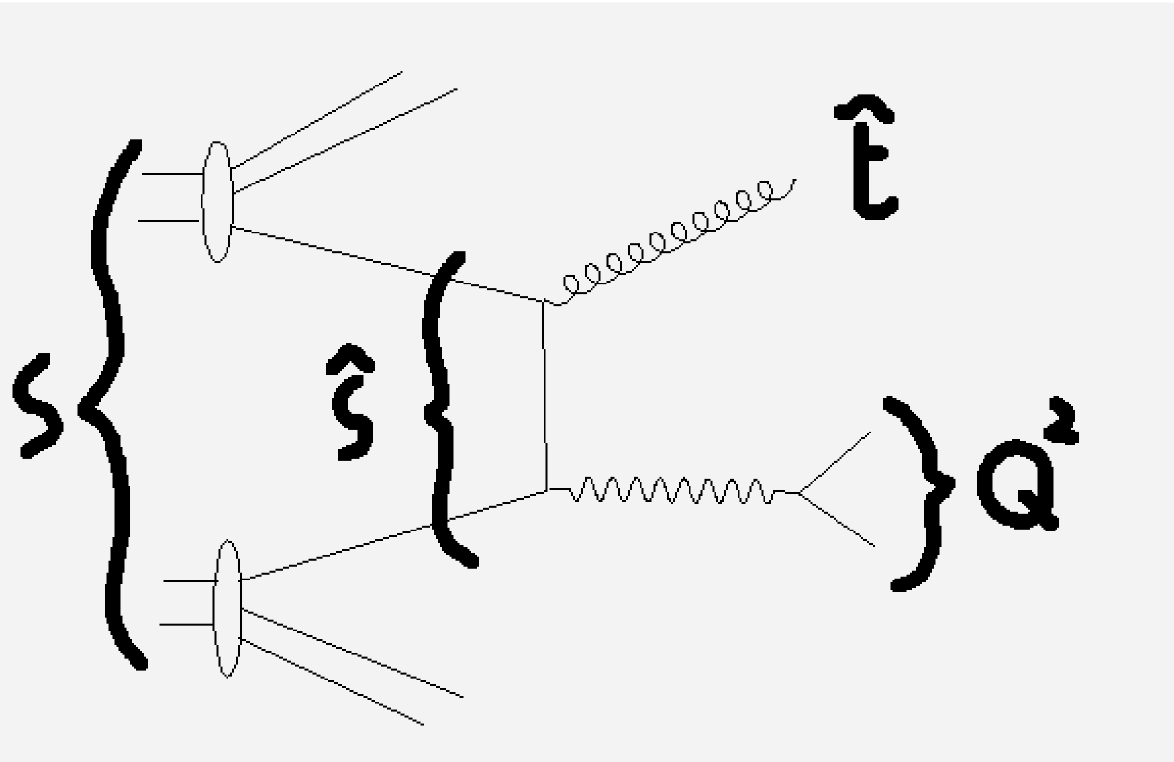
So, all complete diagrams contain two strong vertexes, additional term = $O(\alpha_s)$

Then, some more graphs are present, that differ from these for two vertexes being exchanged etc.

(1): virtual graphs conserve parton model kinematics

(2): for any virtual diagram that may advantage x_1, there is a symmetric graph that equilibrates things

So we expect the virtual correction to be proportional to $G(x_1)G(x_2)$ (by an infinite factor!)



Now, TWO cm-energy transfers

$\hat{S} = x_a x_b\, S$

$Q^2 = x_1 x_2\, S \;=\; \frac{x_1 x_2}{x_a x_b}\hat{S}$

Experimentally we have access to x_1, x_2, not x_a,x_b.

What is missing has gone with $\hat{t}$

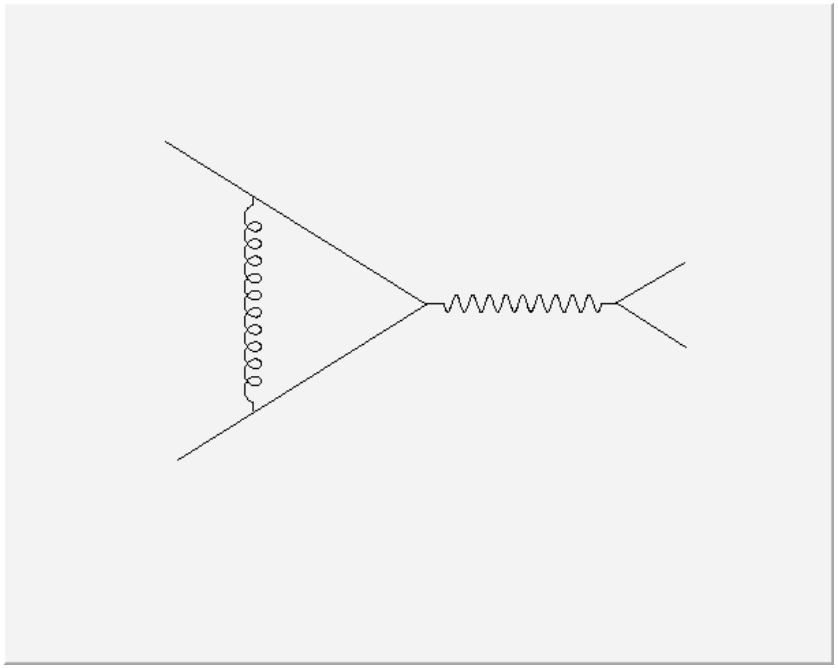
$\hat{t}$ is a function of all the variables we may see in the figure and of q_T^2.

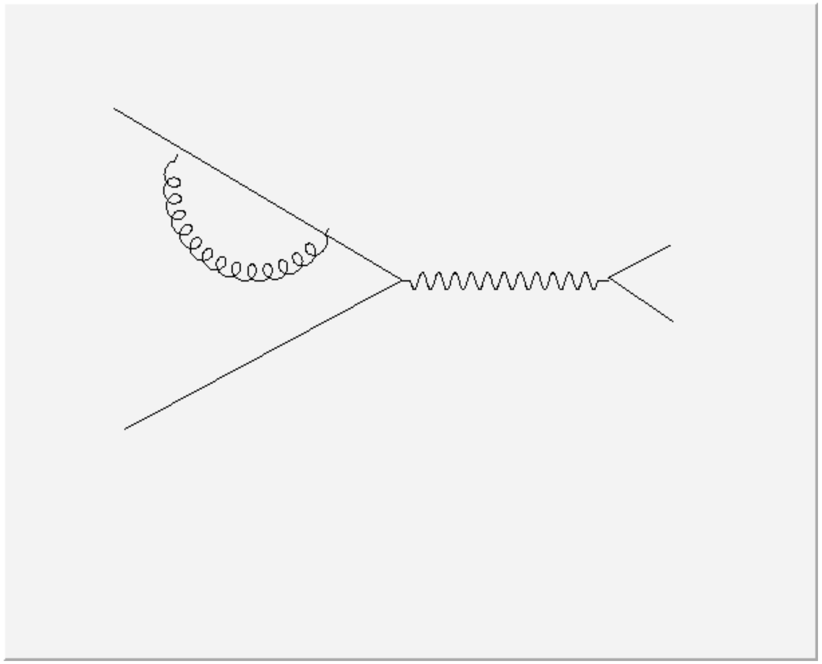
For fixed q_T^2, all the kinematical variables are fixed by Q^2, S, x_1, x_2

$\dfrac{d\sigma}{dQ^2 dq_T^2}$ does not require extra integrations with respect to the parton model case

$\dfrac{d\sigma}{dQ^2}$ requires further integrations, with associated infinities.

At this point, it is important to understand how we remove infinities: the procedure affects deeply the meaning of what we find





The UV divergence of the vertex correction is cancelled by the UV divergence in the self-energy correction (together with its obvious symmetric partner).

Infrared divergencies: they are present in all the diagrams where a gluon is present (real or virtual).

They are caused by gluons with very small, or infinitesimal, energy.

The solution of this problem is known in QED since Bloch-Nordsiek (1937)

What mathematically happens is that in several observables we have cancellation of these divergencies when we sum over real and virtual graphs.

Physically it means that we cannot distinguish between a real and a virtual gluon emission, if its energy is too small.

Collinear divergencies are the most interesting. They arise when a gluon is emitted with a very small angle with respect to one of the colliding fermions.

Since works by Politzer, Radyuskin, Kogut, Soper (1976-77) we exploit the fact that the same collinear divergences are present both in DIS and in DY.

A simple scheme for temporaneous calculation of cross sections

(order 1) is nonzero m_{gluon}

$$\sigma^1 \propto 2\frac{\alpha_s}{2\pi} P_{q\to qg}\left(\frac{x_1 x_2}{x_a x_b}\right) \log(Q^2/m_g^2) \text{ + other relevant terms } \equiv 2\,\sigma^1_{collinear} + \sigma^1_{central}$$

$P_{q\to qg}$ is a collinear QCD splitting function. Its argument tells us how much the (anti)quark x has been decreased by the gluon emission

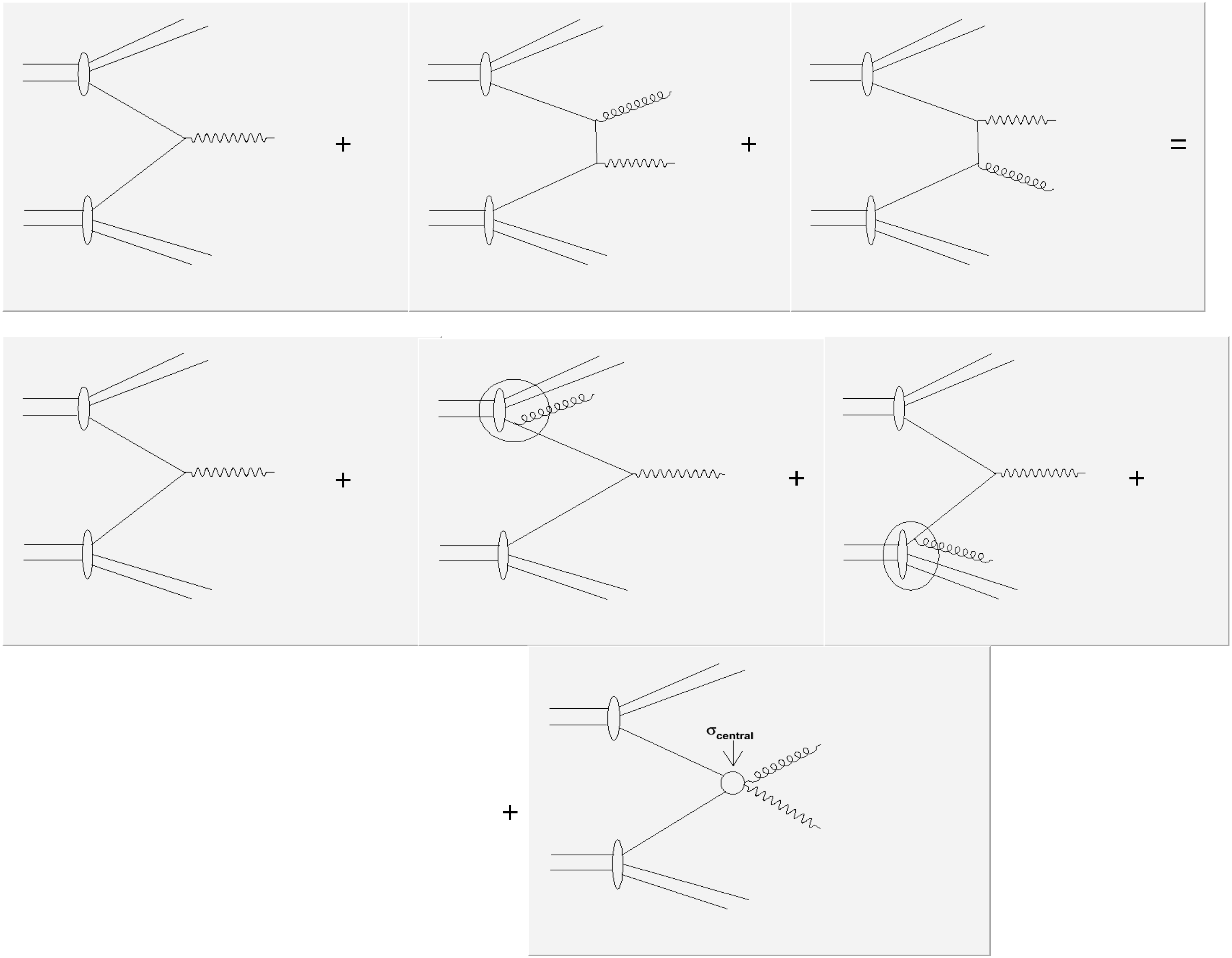
this enters in $\overline{G}(x_2\} G(x_1)\ (\sigma^0 + \sigma^1 + ...)$

one may "extract" the two $\sigma_{collinear}$ from σ^1 and stick them to G and $\overline{G}$:

$$G^0\overline{G}^0(\sigma^0 + \sigma^1) \rightarrow (G^0 + G^1)(\overline{G}^0 + \overline{G}^1)(\sigma^0 + \sigma^1_{central})$$

order 0 (parton model): $G^0G^0\sigma^0$

Order 1: $G^1G^0\sigma^0 + G^0G^1\sigma^0 + G^0G^0\sigma^1_{central}$


$\sigma_{central}$

The same “rearrangement” technique may be applied to the **compton-like** terms, where in one case we have quark*gluon distribution product, in the other one antiquark*gluon distribution product.

Also in this case, the first order cross section may be split into collinear and central terms, and the collinear terms are stick to the distribution functions.

Result: each Drell-Yan fermion distribution function has two first order collinear corrections, coming from real gluon emission and from compton-like diagrams

Real-gluon and compton-like corrections are present in DIS too.

The differences between DY and DIS are relevant, however in the collinear contributions we have the quasi-universal situation of an (almost) on shell quark radiating a gluon.

We may calculate G^1 for the DIS case, as we did for Drell-Yan, and define

$$(G^0 + G^1)_{DY} \equiv (G^0 + G^1)_{DIS} + \text{remains}$$

$$G^0_{DY} = G^0_{DIS}$$

"remains" do not depend on m_g, nor on the chosen regularization scheme

Sometimes the central term is inserted into ``remains", conserving the parton model framework. In such a scheme ``remains" contain regular terms, a delta(1-x), and a (+)-function of x (i.e. x-integrable)

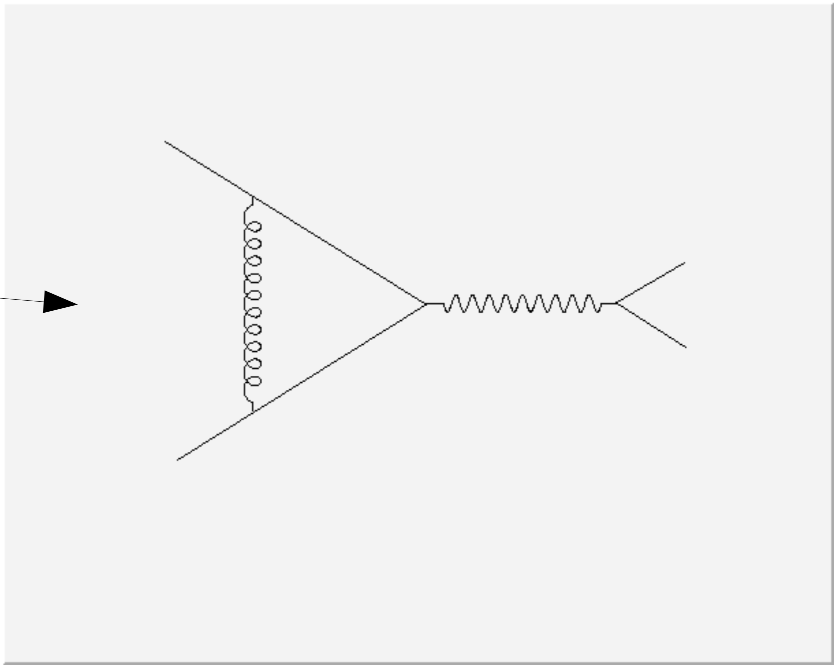
In the DIS case, the cumulative effect of QCD corrections is small

This allows for similarity in the two ways one may calculate the K-factor:

1) $K = \dfrac{\sigma_{DY}}{\sigma^0_{DY}}$ 2) $K = \dfrac{\sigma_{DY}}{\sigma_{DY}(\text{parton model with } G_{DIS})}$

(theoretical way) (empirical way, not good with pions)

For $Q^2/S = (x_1 x_2)$ not too close to 1,
95 % of the K-factor for $d\sigma / dQ^2$
comes from this graph



This is fundamental, for two reasons:

1) it does not touch the parton model kinematics

2) we know the final sum of all the graphs of this family (vertex corrections with one, two, .. exchanged gluons).

Were not for (2), we could never dare trusting the convergence of a perturbative series where the first correction has the same size of the zero order term.

So, the cross section corrections are organized this way:

$$\frac{d\sigma}{dQ^2} = A^0\,(1 + B + C + ...) \quad = A^0(1 + B + ...)(1 + C + ...)$$

``B'' $\rightarrow$ vertex correction, $\approx 2\,\alpha_s$

``C'' $\rightarrow$ all the other $O(\alpha_s)$ terms (small in perturbative regime)

All order vertex sum: $(1 + B + ...) \rightarrow e^B$

So the K-factor is $\quad \exp\left(2\alpha_s(Q^2)\right)(1 + b_1\alpha_s(Q^2) + ...)$

For $(x_1x_2) = Q^2/S$ far from 1 the K-factor almost coincides with

the exp(..) term and b_1 is small.

In a wide range of $Q^2/S = x_1 x_2$, the K-factor is constant

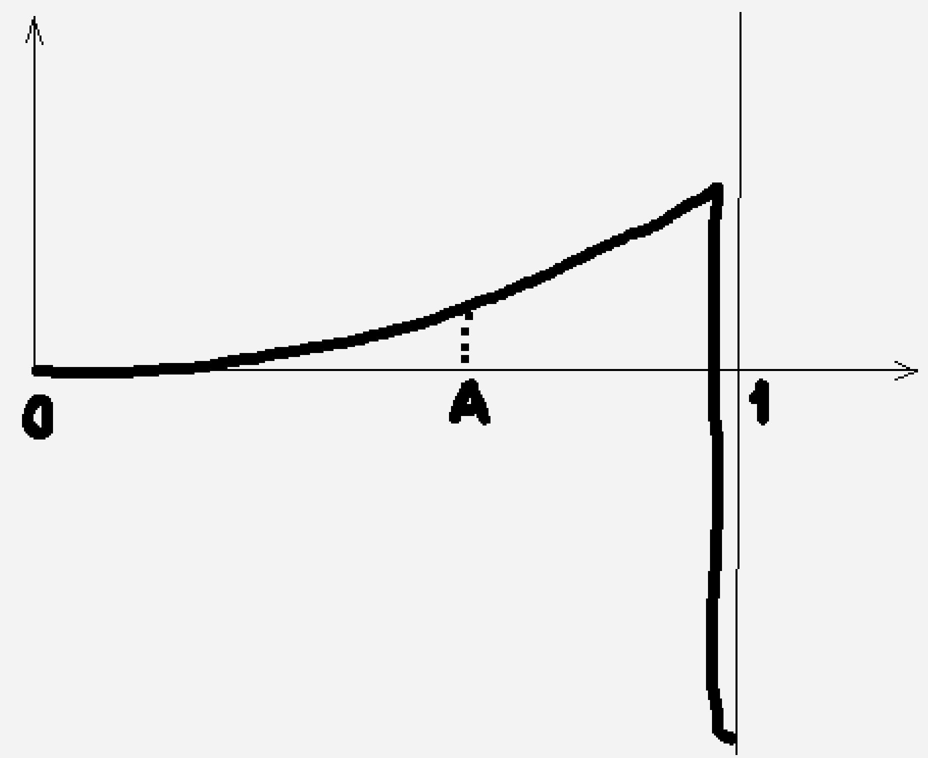
For small Q^2/S, $\alpha_s(Q^2)$ makes the K-factor increase

For Q^2/S coming close to 1 the K-factor increases due to the "remains".

``Remains" contain a (+)-function $f_+\left(\frac{x_1 x_2}{x_a x_b}\right)$

It appears in $\int dx_a\, dx_b \ldots f_+\left(\frac{x_1 x_2}{x_a x_b}\right)$

with integration region $x_1 x_2 < x_a x_b < 1$



The integral of f_+ is zero on the full integration range 0 to 1.

In the range A to 1 it is nonzero,

and increases for A coming close to 1.

These are named “threshold enhancement”, and here are shown at order 1, and affect the K-factor from 0.8 to 1.

In 2005 **Shimizu, Sterman, Vogelsang, Yokoya** have recalculated the effect at all orders.

Their result is a larger threshold effect, with much more visibility at $Q^2/S < 0.8$.

This should affect cross sections at Compass, and in a spectacular way at Panda.

I hope I will live long enough to see whether this is true...

but anyway this would mean that the role of real-gluon production (the main non-virtual contribution to the K-factor) is “more” relevant than estimated up to now on a single-gluon basis.

The “optimistic” scenario could be: the largest part of the corrections are due to a a set of resummed vertex corrections, that do not touch parton model features. At order zero the distribution functions are the same in DIS and DY, and one coud hope that the resummed corrections do not create big differences on this side.

Yoh et al 1978, Ito et al 1981, proton beams

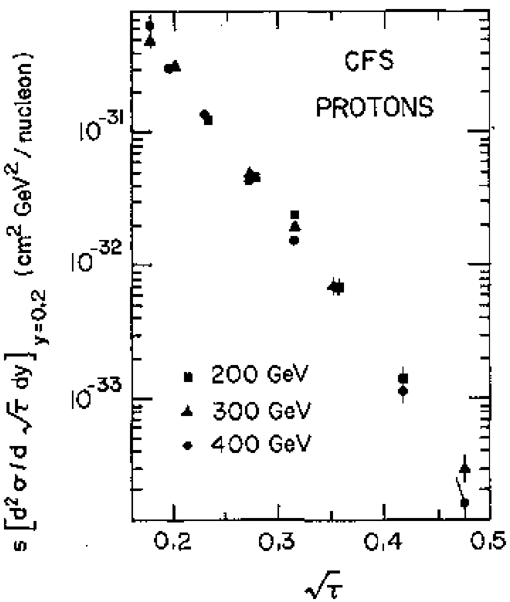


Fig. 10

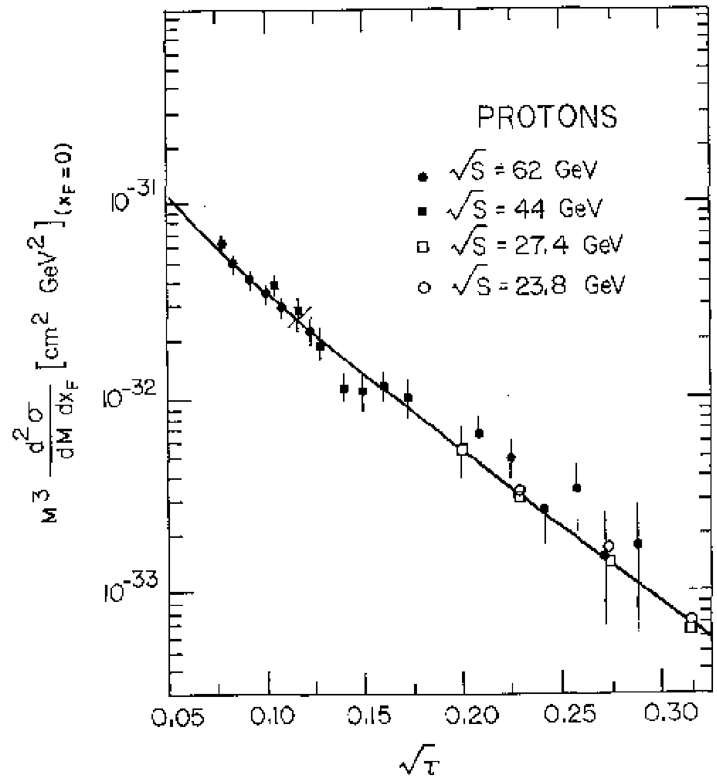


Fig. 11

ISR pp at y=0

Antreasyan et al 1981

Plus CFS data at lower energy.

cm-energy changes by 1.4

Actual cross sections by 10

Small y-range near y=0.2

SCALING CHECKS

“clever” choice: minimizes evolution effects.

Pion beams

Compared NA3 (Lefrancois et al 1980)
and Omega (Corden et al 1980)
for y > 0.
cm-energy scales by 3.2

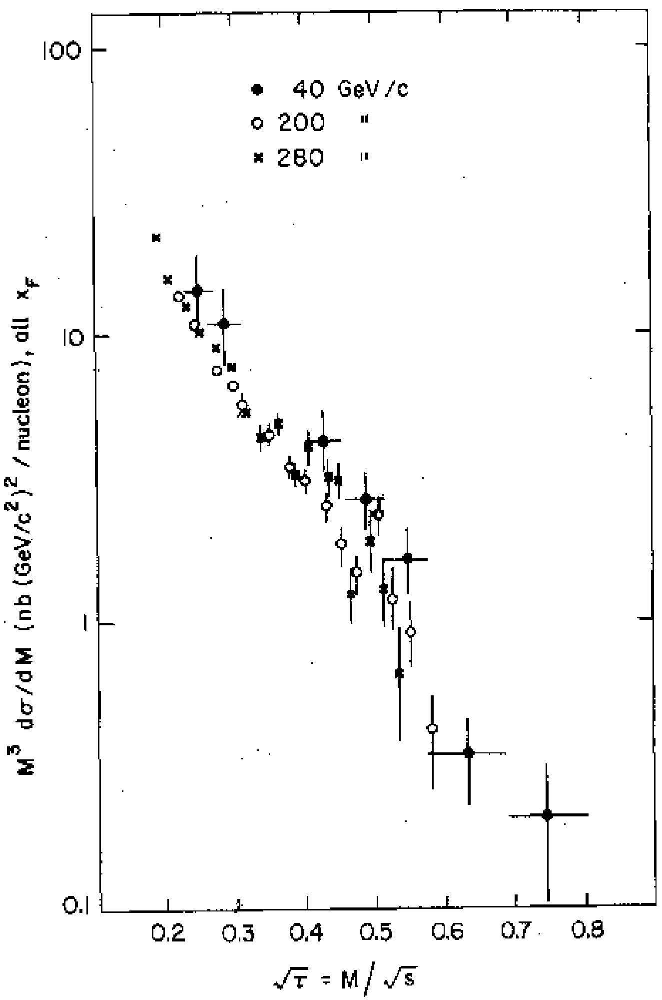


Fig. 12

Compared proton vs negative pion data.

In the sea dominated regions we expect the two cross sections to converge.

In the opposite limit proton data must be suppressed by lack of valence antiquarks

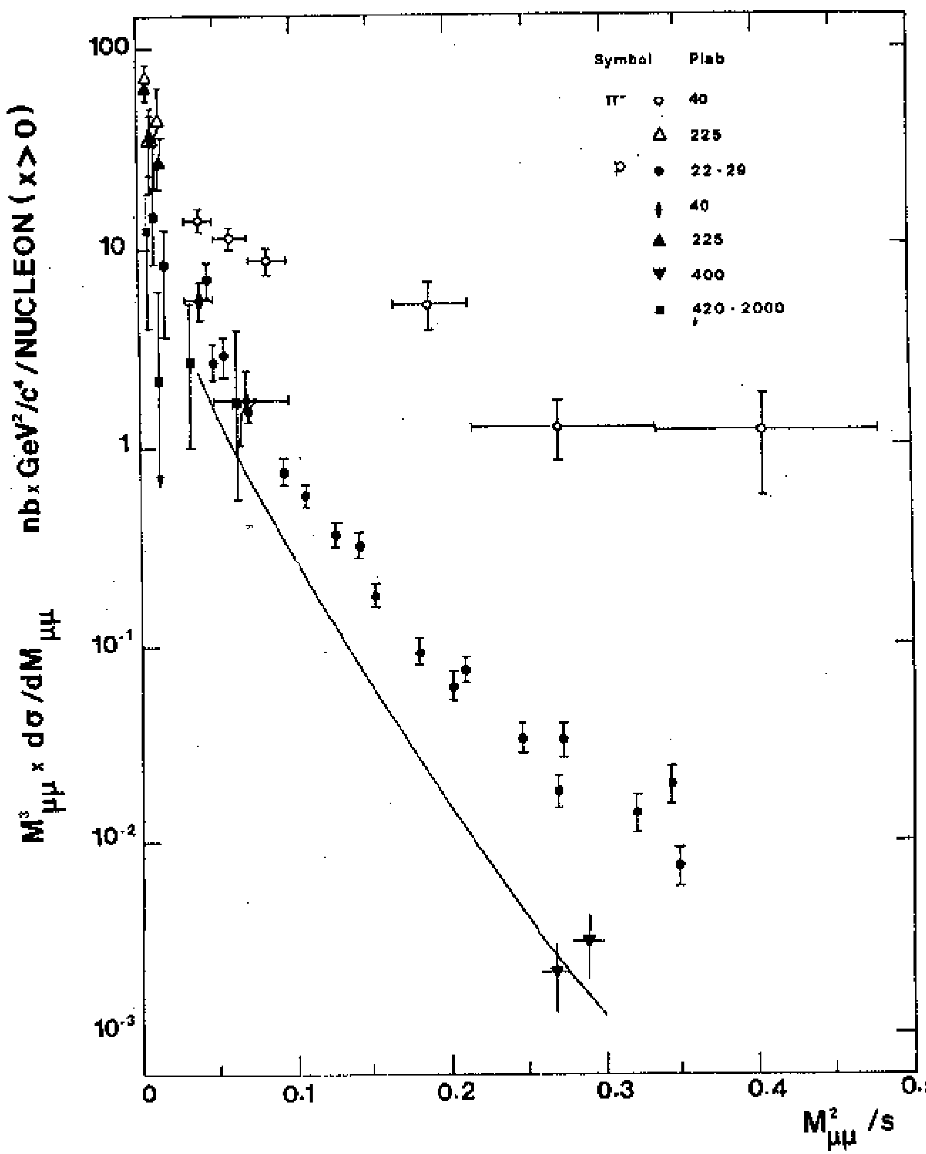


Fig. 13

Positive vs negative pion data.

On isoscalar/proton targets the valence limit of the ratio is 0.25/0.12

Big errors on the pi-plus side

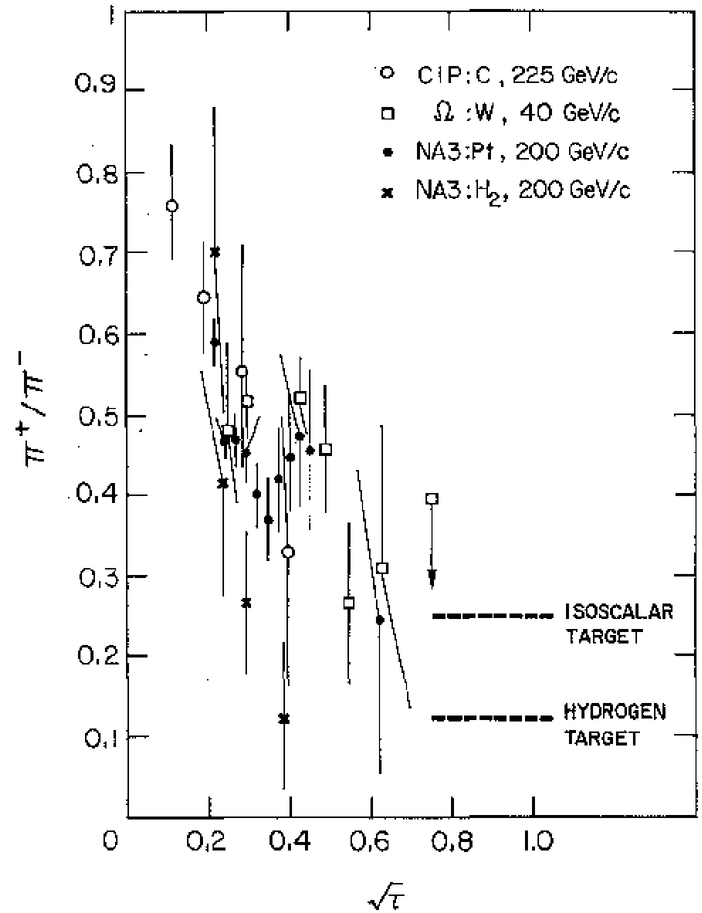
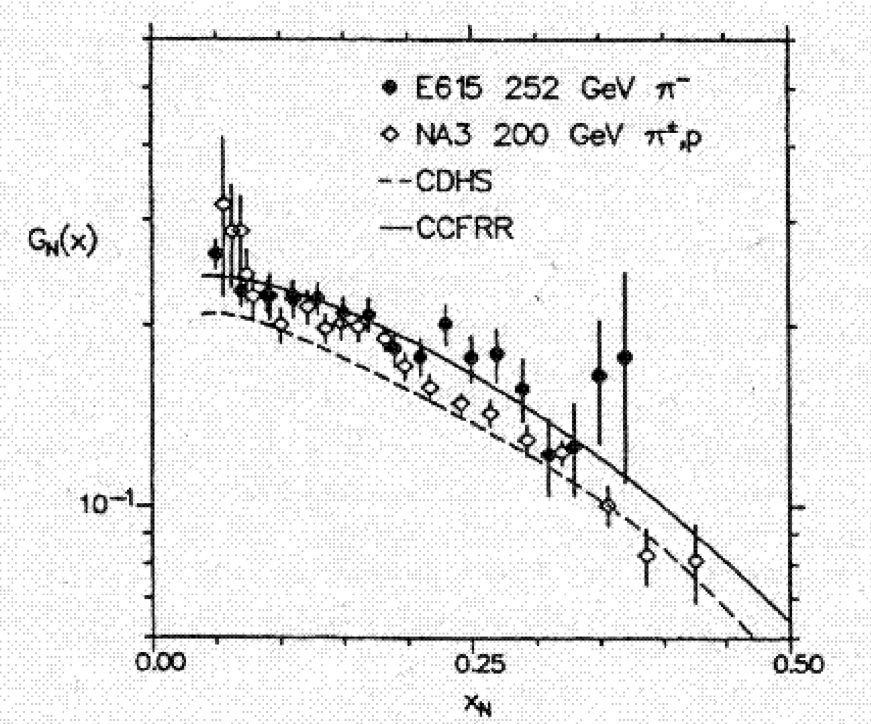
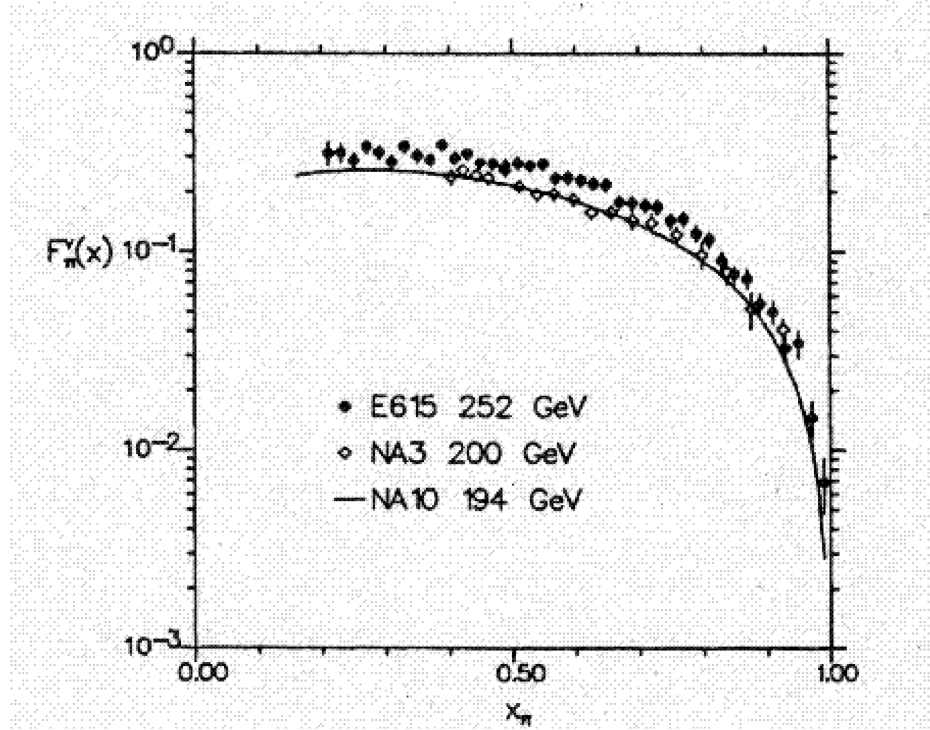


Fig. 15



Nucleon



pion

Here structure functions extracted from pion-nucleus data are compared with those extracted from neutrino-DIS data

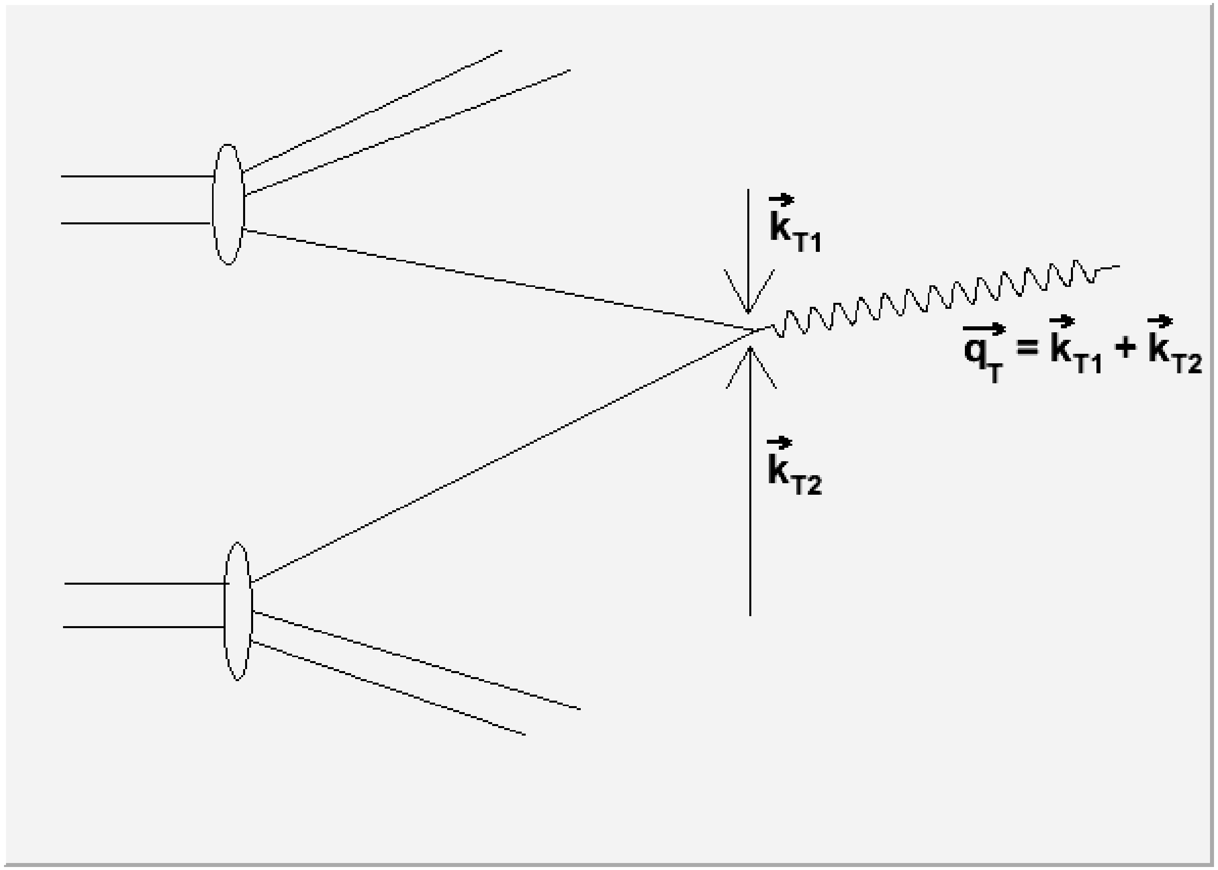
Evidently, there is more precision on the beam-particle side. Notice x-ranges.

Sets like 36,000 (E615), or 155,000 (NA10) dimuons at Q > 4 GeV are used.

Proton-proton DY at high energy machines has reached large statistics, but at advantage of the small-x regions (e.g. E866). The above ones are the last “valence-valence” DY experiments up to now.

Transverse momentum dependence – **Fermi motion scheme**

q_T is a sum of transverse momenta carried by quark and antiquark because of fermi motion



Dominates at $q_T < 0.5$ GeV/c

Gaussian dependence on q_T

$\langle q_T \rangle$ has size $1/R_{hadron}$ → underestimates reality by 2-3 times

These data have been taken with pion beams at 39.5 GeV by Omega (Corden et al 1981)

For q_T up to 2 GeV/c good gaussian shapes within errors.

So, in this range either the Fermi motion scheme is valid, or it must be modified without changing the data shape.

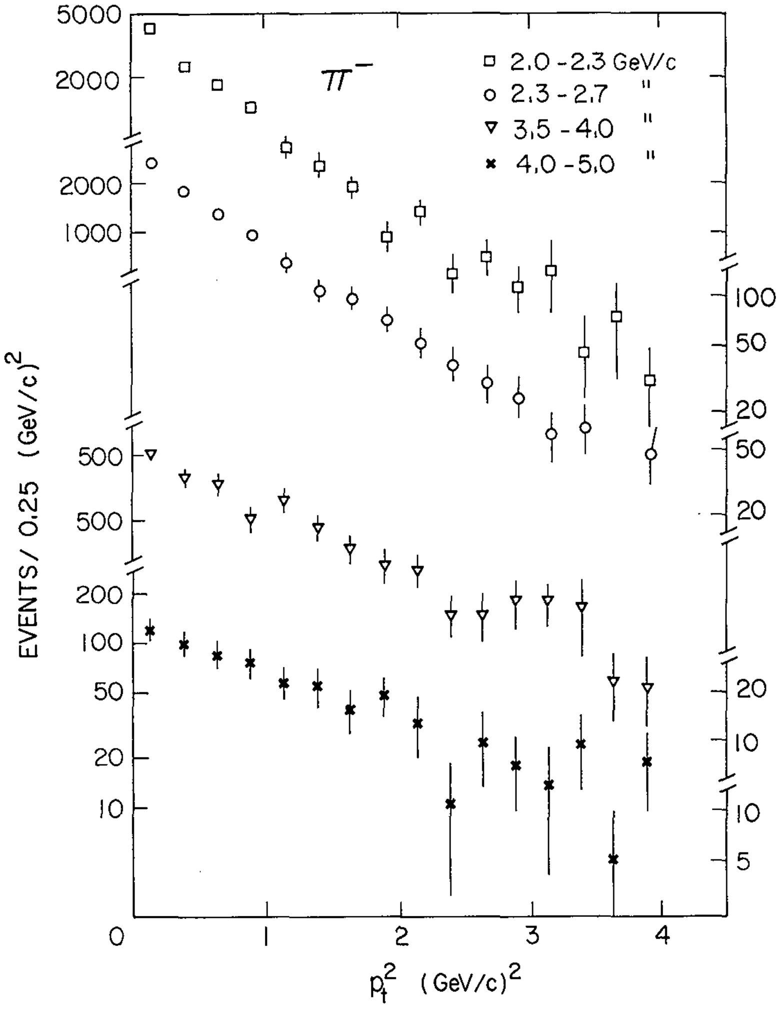


Fig. 6

Transverse momentum:

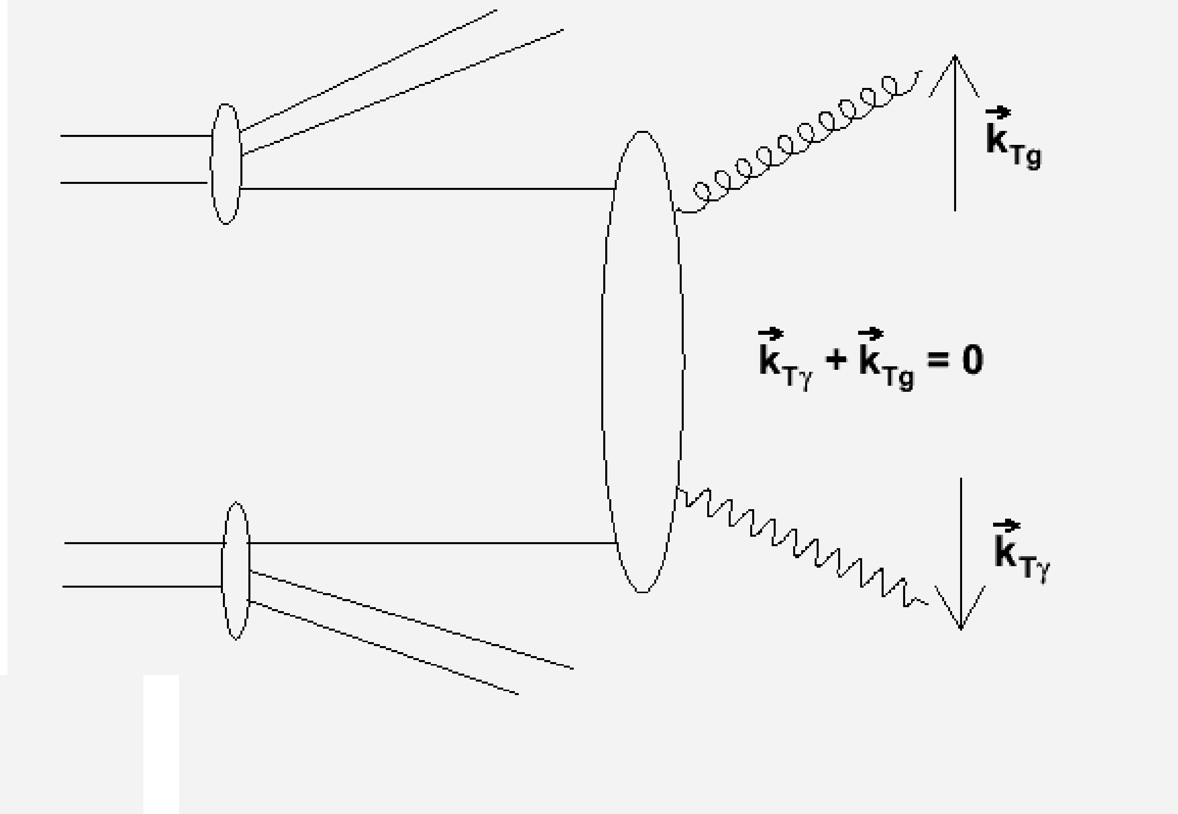
Collinear hard scheme

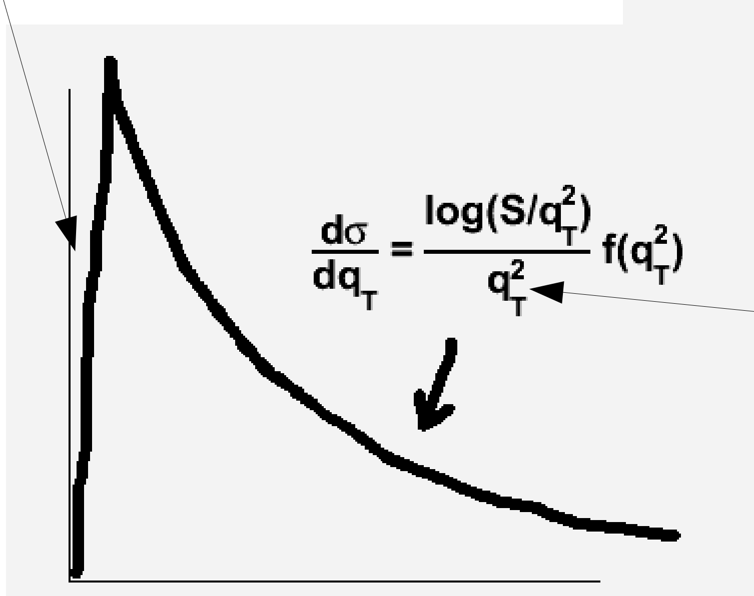
The photon k_T is pure recoil.

The initial quark and antiquark are collinear.

Dominates for q_T >> 1 GeV/c

A Sudakov FF regularizes **very** small q_T (multi-gluon radiation)





$f(q_T)$ is slowly-varying

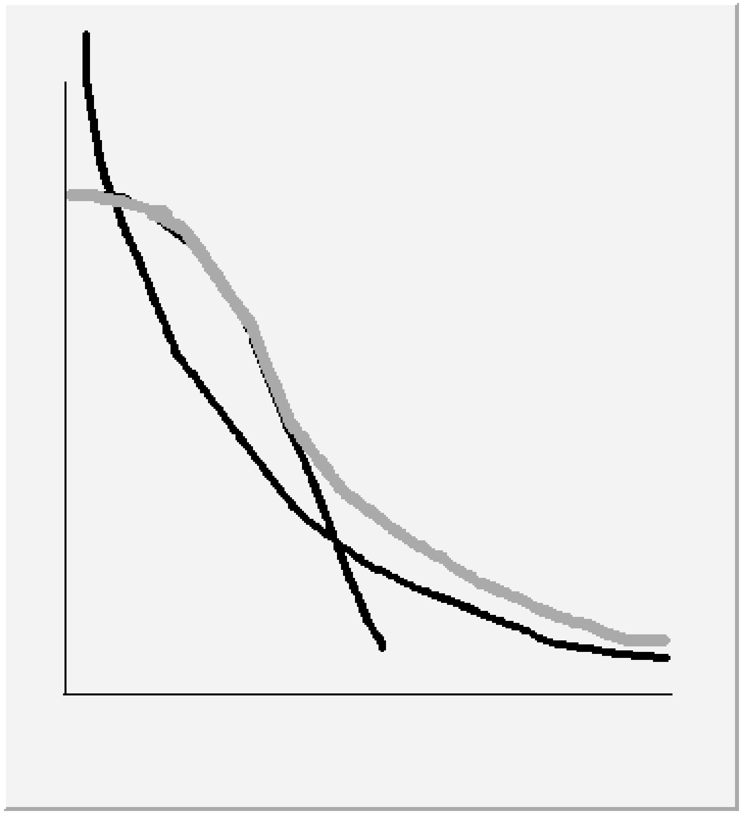
Main factor at large $q_{T:}$

It reproduces the quark-antiquark cross section to photon+gluon.

At increasing q_T **compton** dominates

Transverse momentum

Convolution scheme



It is the obvious merging of the previous schemes, and apart for logical linearity it solves the problem of the quasi-singularity at small q_T.

Because of its quasi-singular peak at small q_T, the collinear term behaves like a delta function respecting the fermi motion shape at small q_T.

However, its application has revealed less straightforward than it may seem, and it underestimates cross sections by a constant factor (about 2).

Several recipes have been developed for this scheme, starting since Altarelli, Ellis, Martinelli, Parisi, Petronzio (1978).

Data from ISR proton-proton
at c.m.energy 62 GeV
(Antreasyan et al 1981)

Fit by Altarelli Parisi Petronzio

with “K-factor” 2

Note: here the role of the compton-like process is enhanced by the large S, meaning small x_1,x_2. , and by the lack of valence antiquarks in the process.

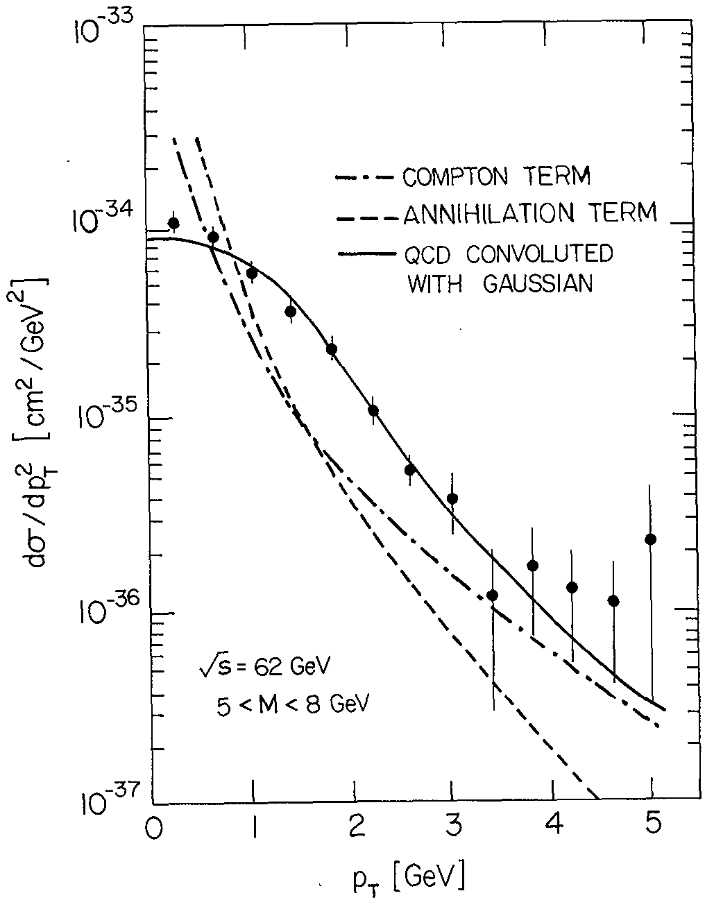


Fig. 23

Data from NA3 with negative pions at 200, 300, 400 GeV beam energy, fixed target (Badier et al, several papers in 1979-81)

Fit by Chiapetta and Greco (1981)

with “K-factor” 1.8

The need for a (large) “K-factor” after calculations that necessarily include QCD contributions rises a doubt:

To which order should one really arrive to reproduce things correctly?

Also, all these points could be reproduced by simple gaussian fits, enlarging the average q_T.

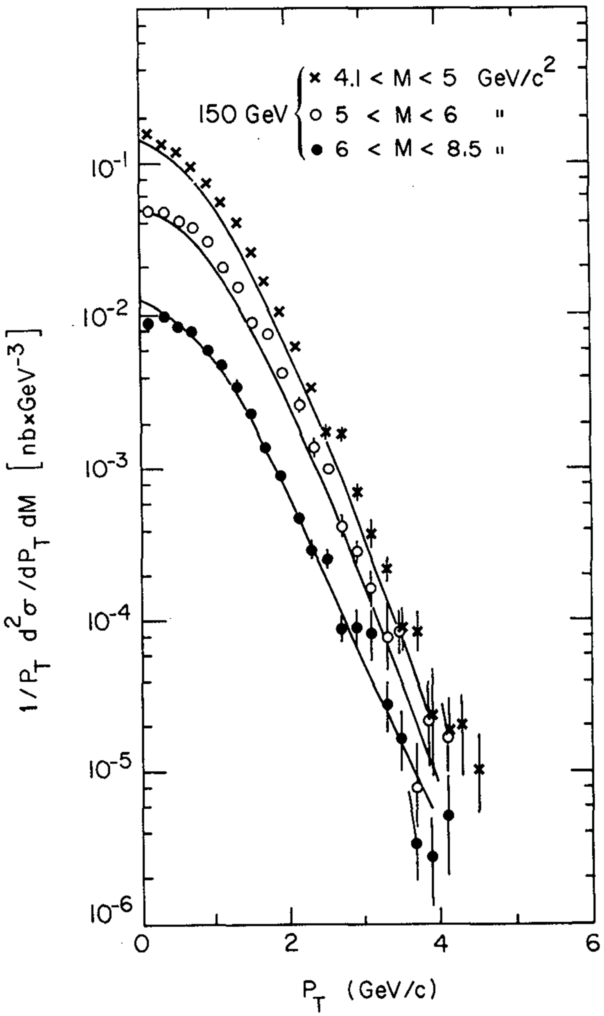
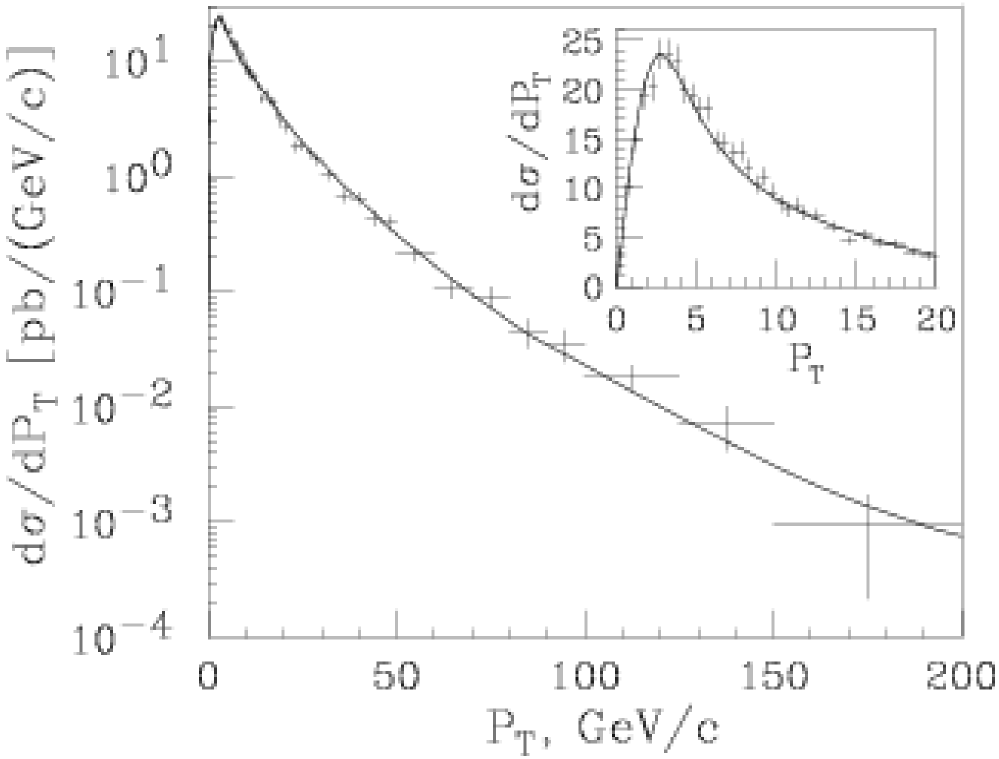


Fig. 24



To enjoy a short trip to Walhalla, here is Drell-Yan q_T spectrum from CDF for 66 < Q < 110 GeV.

Hard QCD features are here well visible:

1) a hard-scale tail that may have nothing to do with fermi motion

2) The fall is not gaussian

3) The Sudakov FF effect is visible at small q_T. It produces a peak at a q_T value that increases with S, so it is hidden by fermi motion smearing for smaller S

There are other observables that could discriminate QCD-peculiar features at Midgard level.
E.g. the way $<q_T>$ depends on S below (left) has nothing to do with Fermi motion. For the dependence on Q (M in the figure) things are less clear.

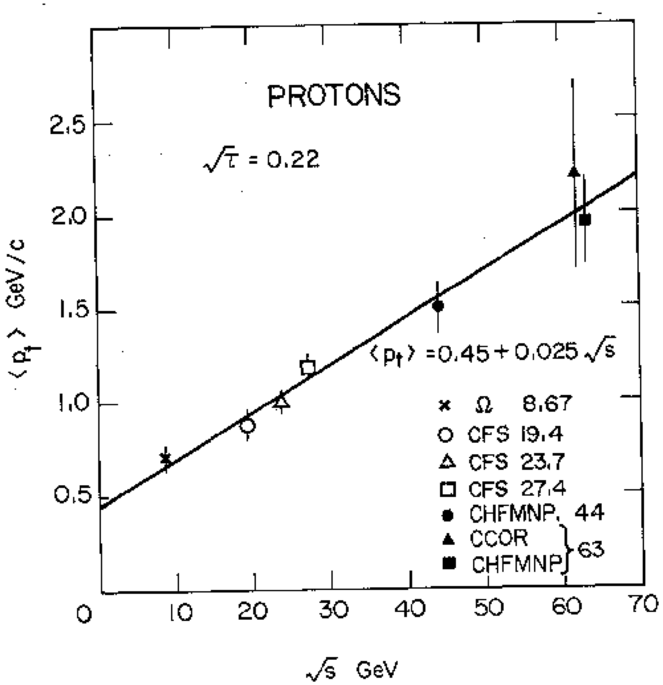


Fig. 20a

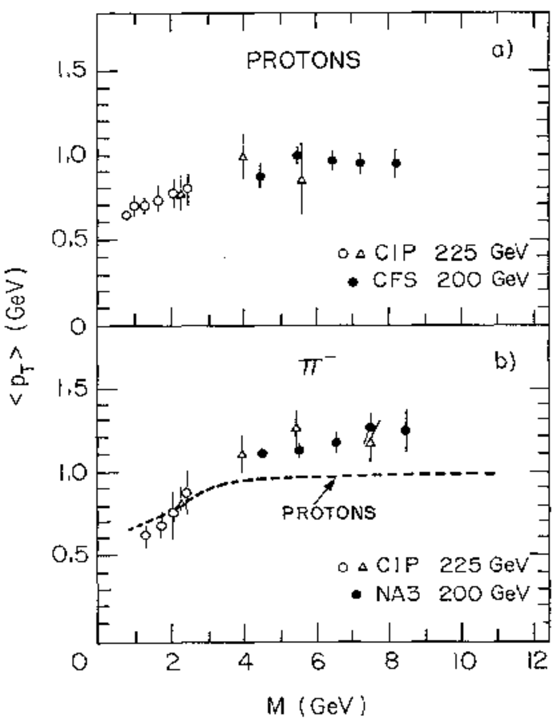


Fig. 18

EXPERIMENTAL STUDY OF MUON PAIRS PRODUCED BY . . .

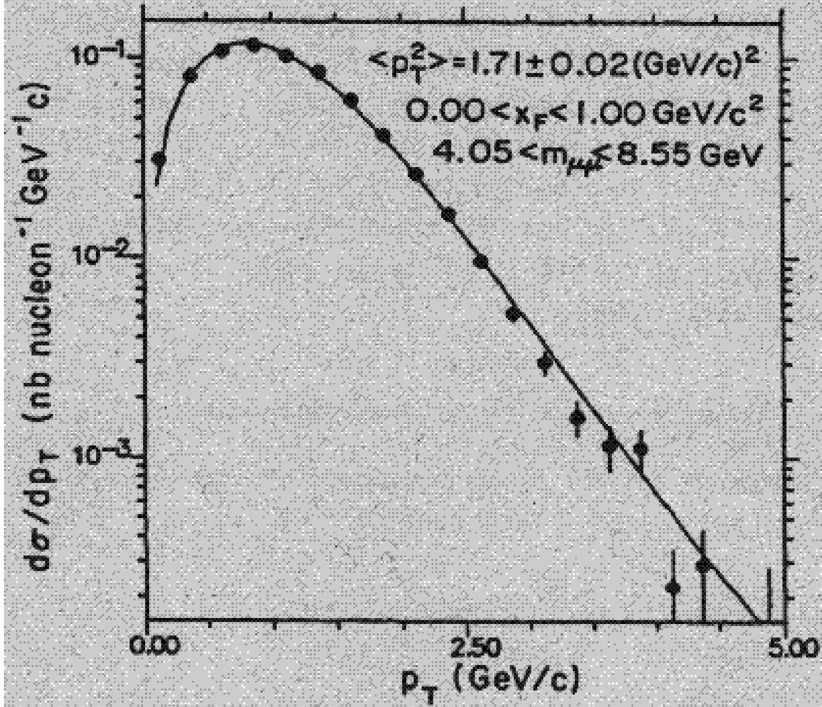
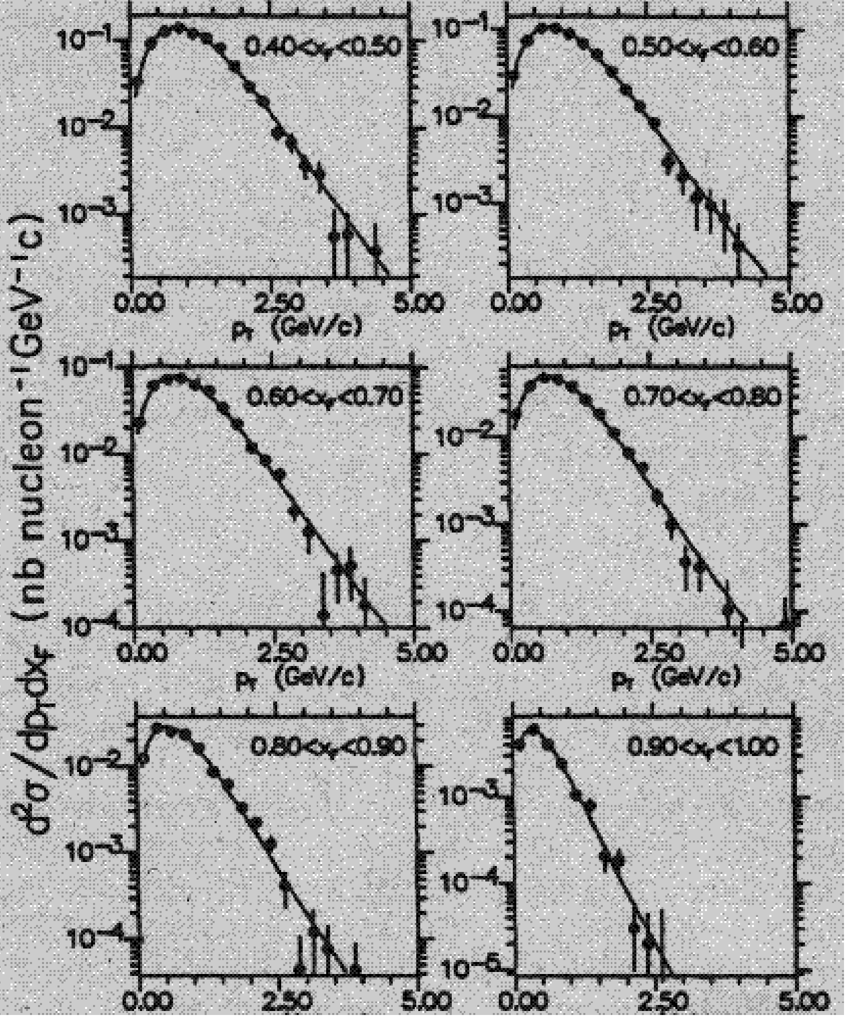


FIG. 23. p_T spectrum corrected for acceptance. The back-
ound, as estimated from random pairings, is negligible.

together with the parametrized fit. As can be seen, the
ality of the fit is excellent.



A mystery: what happens at $q_T < 1$ GeV/c? These data are from E615 (Conway et al 1989), pions at 252 GeV on W. This “small”-q_T behavior appears only here (but this experiment has far the finest q_T-scanning among the pion ones). At this S the Sudakov FF effect cannot be trivially expected to be visible.

Nuclear dimuon cross sections fitted as $\mathbf{A}^{\alpha}$

This is another mystery:

where is the nuclear dependence?

Here the parton model is TOO MUCH good.

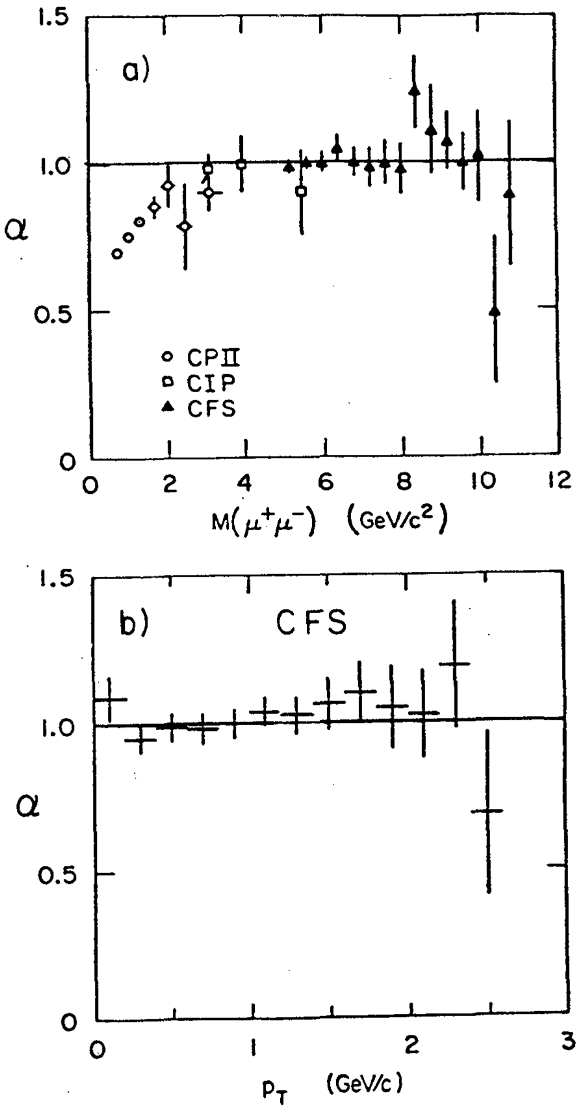


Fig. 16